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# Permeability Estimation Based on Pore Radius and Its Distribution

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***Abstract***
*Hydraulic permeability may be expressed as a function of pore radius. A simple capillary model shows that permeability is proportional to the square of pore radius. This model may be extended by including pore size distribution, so statistical parameters of its pore radius distribution can be considered in permeability estimation. Another recent approach, the so-called effective medium approximation uses pore radius distribution as an input parameter for permeability calculation.*

*Permeability can be estimated with the help of the above approaches as far as the pore radius and its distribution are available. Pore radius distribution can be generated using digital image processing. By defining pore radius as the ratio of image pore area and its circumference, pore size distribution of rock image can then be created.*

*Estimation of permeability by means of the effective medium approximation gives better results than the simple capillary model as well as its extended formulae. Definition of pore radius has however significant influence on the results of estimation, since different definitions of pore radius produces different distributions.*

***Keywords****: permeability, pore radius, capillary model, EMA.*

***Abstrak***
*Permeabilitas hidraulik dapat dinyatakan sebagai fungsi radius pori. Model kapiler sederhana memperlihatkan bahwa permeabilitas sebanding dengan kuadrat radius pori. Model ini dapat dikembangkan dengan memperhitungkan distribusi radius pori, sehingga parameter statistik distribusi pori tersebut dapat dipertimbangkan dalam estimasi permeabilitas. Pendekatan lain yang baru adalah pendekatan medium efektif yang menggunakan distribusi radius pori sebagai parameter masukan untuk penghitungan permeabilitas.*

*Permeabilitas dapat diestimasi dengan memanfaatkan pendekatan-pendekatan di atas jika diketahui radius pori dan distribusinya. Radius pori dapat diperoleh dengan bantuan pemrosesan citra digital. Dengan mendefinisikan radius pori sebagai rasio antara luas dan keliling pori citra, maka distribusi radius pori sebuah citra dapat dibuat.*

*Estimasi permeabilitas dengan pendekatan medium efektif memberikan hasil yang lebih baik dari pada model kapiler sederhana maupun turunannya. Akan tetapi definisi radius pori mempunyai pengaruh yang sangat signifikan pada hasil estimasi, karena perbedaan definisi radius memberikan distribusi yang berbeda.*

***Kata kunci****: permeabilitas, radius pori,, model kapiler, EMA.*

## 1. Introduction

Hydraulic permeability ($k$) is one of the very important parameters in reservoir engineering, for example as an input parameters for managing oil, gas and water reservoirs. In environmental geophysics and nuclear waste disposal, information about permeability is also very crucial.

Unfortunately this parameter $k$ cannot be obtained easily. If cores are available, then $k$ can be measured directly using standard method. However, that is not always the case, since it is expensive. Therefore, permeability prediction of real rocks is a very important problem. For that reason, study of pore geometry that controls permeability of porous media remains an active research field, both theoretically and experimentally.

Experimental and empirical study of permeability prediction have been reviewed by several authors[1,2]. Lists of references will be too long to write here. As well as experimental, theoretical study is also conducted intensively by authors. Sahimi[3], for example, gives a review of some theoretical approaches. The application of the recent theories and approaches on real rocks especially for permeability estimation in several cases work better than simple empirical relations[4].

Digital image processing is helpful to quantify microgeometry of rock. Digital image equipments for characterization of rock have been developed several years ago. Several authors applied digital image processing for studying microgeometry of rocks. Rink and Schopper[5] used mechanical system and photo-multiplier to create digital image from rock samples. CCD camera was used more intensive to produce digital image of rock. Scanning electron microscope (SEM) was also applied to analyze rock samples[6]. Three-dimensional reconstruction of rock was carried out by means of laser confocal microscopy (LSCM)[7]. In this paper we used image processing technique to create pore radius and its distribution. The pore radius is then used as an input parameter for permeability estimation. Simple capillary models and effective medium approximation (EMA) were applied in this research.

## 2. Theoretical Background

The fluid volume flow Q in a simple capillary model can be described by Hagen-Poiseulle law[8]:

$$Q = \frac{\pi r_k^{\,4}}{8\eta}\frac{\Delta P}{l} \tag{1}$$

where $r_k$ is the capillary radius, $l$ is the length of the capillary, $\eta$ is the dynamic viscosity and $\Delta P$ is the pressure different. Combining with the Darcy's law for fluid flow we arrived to the following equation:

$$k = \frac{\phi r_k^{\,2}}{8\tau_h} \tag{2}$$

here $\phi$ is porosity und $\tau_h$ is the so-called hydraulic tortuosity that is the ratio between complex path flow and its shortest path. For an arbitrary cross section of capillaries, equation (2) may be modified by introducing shape factor $f$:

$$k = \frac{\phi r_k^{\,2}}{4 f \tau_h} \tag{3}$$

The shape factor has the value between 2 and 3 as can be seen below :

Table 1. Shape factor according to Carman[9].

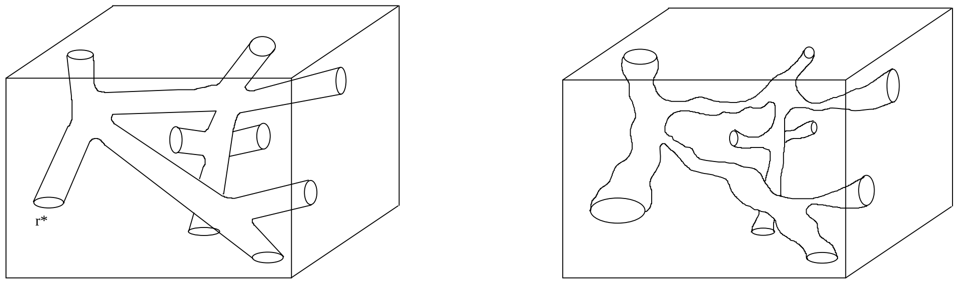
| Cross section | | Shape factor $f$ |
|---|---|---|
| Cylindrical | a/b = 1 | 2.00 |
| Elliptical | a/b = 2 | 2.13 |
| | a/b = 10 | 2.45 |
| | a/b = 50 | 2.96 |
| Rectangular | a/b = 1 | 1.78 |
| | a/b = 2 | 1.94 |
| | a/b = 10 | 2.65 |
| | a/b = ∞ | 3.00 |
| Triangle | | 1.67 |

Equation (3) may be extended by considering pore size distribution[10], i.e.:

$$k = \frac{\overline{r_k}^{2}\ \phi}{4 f \tau_h}\left[\frac{(\gamma C_k^3 + 3C_k^2 + 1)^2}{(1 + C_k^2)^2}\right] \tag{4}$$

where: $\overline{r_k} = \int_0^\infty r_k\, f(r_k)\, dr_k$ , $f(r_k)$ is the distribution function of the pore radius, $C_k$ is the variation coefficient $= \frac{\sigma_{dev}}{\overline{r_k}}$, $\sigma_{dev}$ is the standard deviation, and $\gamma$ is the skewness = $\frac{1}{\sigma_{dev}^{\;3}}\int_0^\infty (r_k - \overline{r_k})^3\ f(r_k)\, dr_k$ .

The above-mentioned capillary models assumed that rock pores are parallel without any connections. Coordination number that is the number of capillaries meeting in a node is not yet considered. Rock is a complex system containing connectivity among capillaries. The connectivity may be included in the calculation with the help of effective medium approximation (EMA). The EMA approaches the complex pore system to a homogenous pore system with an effective radius as illustrated in Figure 1:




a. Inhomogeneous real rock b. homogeneous pore system

Figure 1. Illustration of EMA[11].

Permeability can then be calculated using the following formula[11]:

$$k = \frac{\phi}{8\tau_h} \frac{(r_h^*)^4}{< r^2 >} \quad (5)$$

The effective radius $r_h*$ should be obtained by solving EMA equation as follows[12]:

$$\int f(g) \frac{g_{eq} - g}{g + (\frac{z}{2} - 1) g_{eq}} dg = 0 \quad (6)$$

$g$ is denoted to hydraulic conductivity, $z$ is the coordination number, i. e. number of throats meeting in a node. Equation (6) is then solved iteratively to get $r_h*$. It is still not easy to obtain the coordination number of real rocks. Several authors developed therefore some techniques to estimate the coordination number. In some cases, the coordination number of highly porous rocks is sometimes set to 6[11]. Dullien[13] described an empirical relationship between porosity and coordination number:

$$1 - \phi = 1.072 - 0.11937z + 0.0043z^2 \quad (7)$$

Some experimental results show that coordination number increases with increasing porosity.

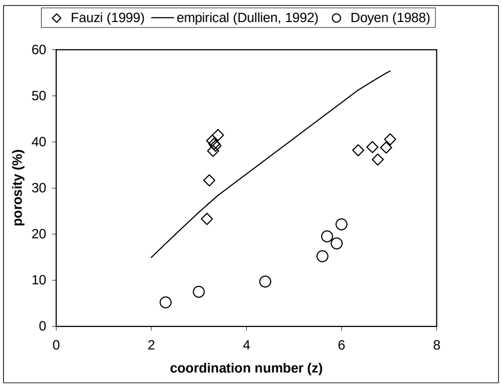



Figure 2. Experimental result and empirical relationships of coordination number as a function of porosity.

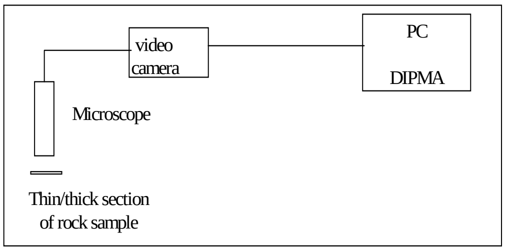
The solid line is the empirical formula given by[13]. The circle is the coordination number estimated from image section[11]. The square is the result from “toy’s rock” after Fauzi [14].

**3. Experimental procedures**

First of all, thick or thin sections are produced. Since we would like to analyze the microstructure of the rocks and not the minerals itself, thick sections are more appropriate. Controlling the thickness of the section may produce good quality image.

Images are created from thin or thick sections of rock samples with the help of a microscope and video camera, which is connected directly to the personal computer. Software DIPMA (Digital Image Processing for Micro-Analysis), developed by the author and his group is used to process digital image data. The color images are then changed to binary images (black-white images) after choosing an appropriate threshold. Hue-Luminance-Saturation is applied to create the black-white images.

The image analysis procedure and an example of image section are given in Figure-3.



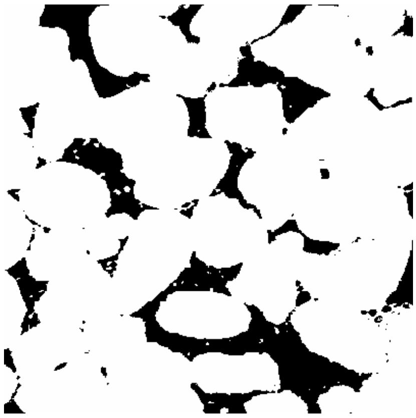
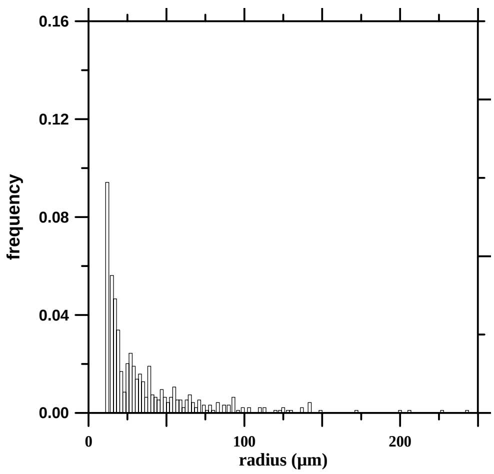
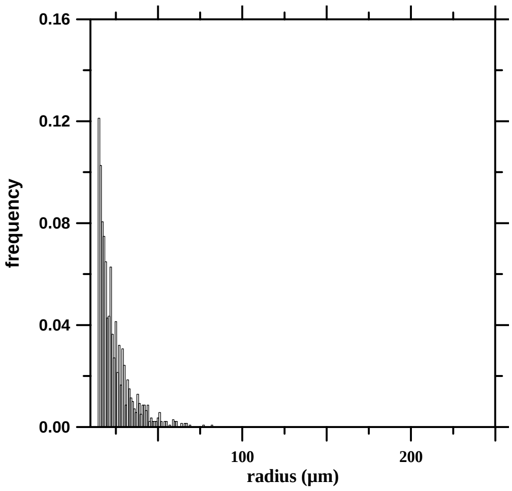



Figure 3. Image analysis for rock characterisation and an example of black-white image. Black represents for pores and white for grains.

Definition of pore radius is crucial in this case, since the pore shape is irregular. Pore radius may be defined according to Wadell, where the cross-section of the pore is assumed of circular shape: $r = \sqrt{\frac{A}{\pi}}$, where A is the area of the pore. Figure-4 shows examples of pore size distribution.




4a



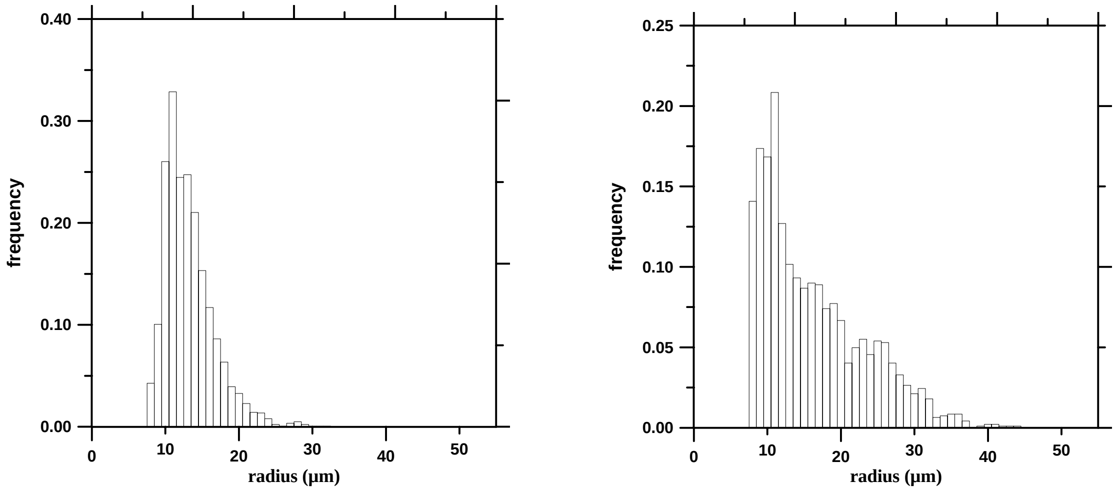

4b

Figure 4. Pore radius distribution of rock sample according to Wadell's definition, a is medium and b is fine sandstone respectively.

In the above definition purely circular shape is assumed. Most of pore shapes however have arbitrarily shape instead of circular. The arbitrary pore shape may be considered by defining the pore radius with the following formula[15)]:

$$r = \frac{2A}{L_A} \tag{8}$$

A : area of the pore, $L_A$: circumference of the pore. Figure 5 is an example of pore size distribution with the pore radius defined as in equation (8).




5a 5b

Figure 5. Pore radius distribution of rock sample according to equation (8), 5a is medium and 5b is fine sandstone respectively.

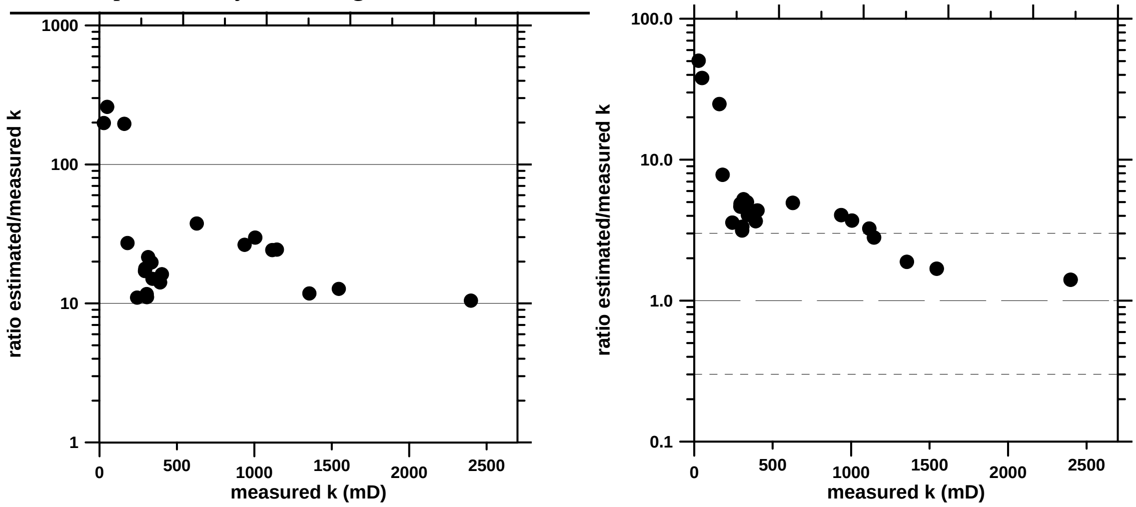
Both definitions of pore size are more suitable for sedimentary rocks without any cracks. Since our samples are sedimentary rocks and loose sands both definitions are therefore appropriate.

It is quite clear that definition of pore size will influence the type of the distribution. It seems that most of our rock samples show log normal distribution as also mentioned by several authors.

## 4. Result and Discussion

Estimation of permeability using the capillary model is shown in figure 6. Figure 6a is the result of a permeability calculation where the Wadell's radius is used. Ratio of calculated and measured permeability is still high. Most of them are higher than two decades. The ratio is higher for the smaller permeability. Although this high tendency was also observed by other authors[16], however it still can be improved by considering the shape of pores radius. The result of permeability estimation using capillary approach, where the pore radius is defined by equation (8) gives closer values to the measured permeability. Figure 6b shows the result of estimations. Most of the ratio between estimated and measured is less than one decade.

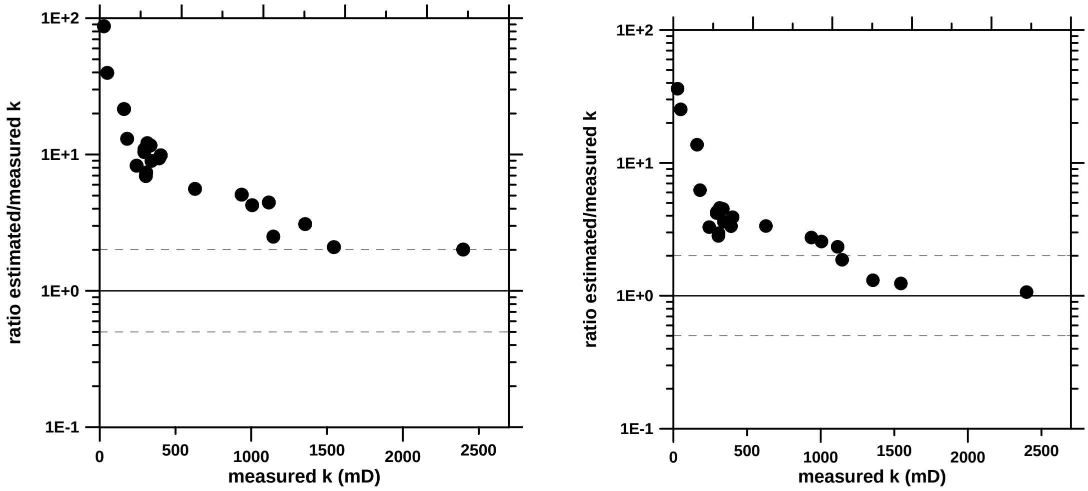
Consideration on standard deviation and skewness of distribution give only minor change on the estimation of permeability in which permeability is expressed as a function of grain size[17]. As the formula for permeability is similar, it will not be explained here.




6a 6b

Figure 6. Ratio of estimated and measured permeability for different definition of pore radius, 6a uses Wadell's definition and 6b equation (8) respectively.

Another approach for permeability estimation with the help of pore radius is the EMA. The EMA has significant difference to the first approach. In the above approaches, coordination number, i.e. number of throats meeting in a node, was not considered. Here we tried to apply the EMA to estimate permeability with pore radius distribution as input parameters. For the first approach, the coordination number is set to be 6, since our samples are sandstone with quite high porosity. The result of estimation using EMA are shown in figure 7.




7a 7b

Figure 7. Ratio of EMA estimated and measured permeability for different of pore radius definition. Figure 7a is using Wadell's definition and 7b is using the definition described in equation (8).

Most of the samples show that the ratio between estimated and measured permeability are less than one decade. The higher the permeability of the samples the closer is the estimated permeability to the measured. Again we see that different definition of pore radius gives different result of estimation.

Comparison of the permeability estimation by means of capillary models and EMA with the same pore radius definition is shown in figure 8.

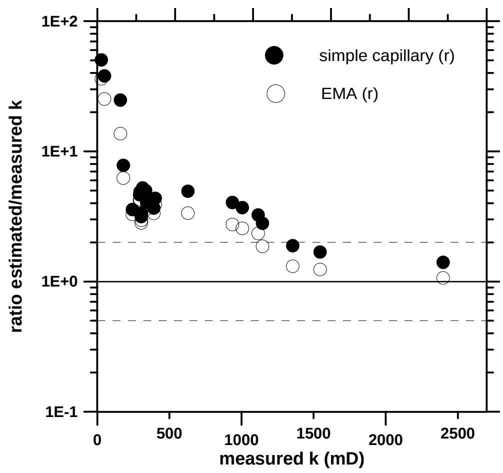



Figure 8. Comparison of estimated and measured permeability using capillary model and EMA.

It is shown that the EMA gives better results than the capillary models. In general the permeability estimation is close to the measured permeability for samples with the permeability more than 250 mD. The ratio of estimated and measured permeability for samples with low permeability, i. e. smaller than 250 mD, is still high, perhaps due to high coordination number.

## 5. Conclusion

In this paper, we used pore radius as input parameter to estimate permeability of rock samples. Different definition of pore radius gives different result of estimation. Definition of pore radius considering not only its area but also circumference gives better result. Estimation of permeability using EMA is closer to the measured permeability than capillary models.

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**References**


1. Sen, P. N., Straley, C., Kenyon, W. E., and Wittingham, M. S., *Surface-to-volume ratio, charge density, nuclear magnetic relaxation, and permeability in clay bearing sandstones*, Geophysics, **55**, 61-69, 1990.
2. Nelson, P. H., *Permeability-porosity relationship in sedimentary rocks*, The log Analyst, 38-62, May-June 1994.
3. Sahimi, M., *Flow and transport in porous media and fractured rock,* VCH Verlagsgesellschaft mbH. Germany 1995.
4. Fauzi, U., Vozoff, K., Neubauer, F. M., Hoerdt, A., *Zusammenhaenge zwischen hydraulischer Permeabilitaet und elektrischer Leitfaehigkeit*, in Bahr, K. and Junge, A., Protokol ueber das 15. Kolloquium 'Elektromagnetische Tiefenforschung', Hoechst, 1994.
5. Rink, M. and Schopper, J. R., *On the application of image analysis to formation evaluation*, 5th European Logging Symposium Transaction. Paris, 1-22, 1977.
6. Berryman, J. G. and Blair, S. C., *Kozeny Carman relations and image processing methods for estimating Darcy's constant*, J. Appl. Phys., **62**, 2221-2228, 1987.
7. Fredrich, J. T., Menendez, B., und Wong, T. F., 1995, Imaging the pore structure of geomaterials: Science, **268**, 276-279.
8. Landau, L. D., and Lifschitz, E. M., *Fluid mechanics*, Pergamon Press LTD, Oxford, London, Paris, Frankfurt, 1963.
9. Georgi, D. T., and Menger, S. T., *Reservoir quality, porosity and permeability relationship*, in Dresen, L., Fertig, J., Rüter, H. and Budach, W. (ed.), Stratigraphie und Lithologie in der geophysikalischen Exploration, 12th Mintrop Seminar, Münster, 1994.
10. Fauzi, U., *Untersuchungen zur Charakterisierung der Porengeometrie von Gesteinen zur Abschätzung der hydraulischen Permeabilität and des Formationsfaktors mit Hilfe von Bildanalysen*, Dissertation, Institut für Geophysik und Meteorologie, Universität zu Köln, Germany, 1997.
11. Doyen, P. M., *Permeability, conductivity and pore geometry of sandstone*, J. Geoph. Res. **93**, 7729-7740, 1988.
12. Kirkpatrick, S., *Percolation and Conduction*, Rev. Mod. Phys., **45**, 574-588, 1973.
13. Dullien, F. A., *Porous media: Fluid transport and pore structure*, San Diego, Academic, New York, 1992.
14. Fauzi, U., *Influence of coordination number on permeability estimation*, proceedings of HAGI annual meeting, Surabaya, Indonesia, 1999.
15. Kulenkampff, J., *Die komplexe elektrische Leitfaehigkeit poroeser gesteine in Frequenzbereich von 10 Hz bis 1 MHz – Einfluesse von Porenstruktur und Porenfuellung*-, Dissertation, TU-Clausthal, 1994.
16. Koplik, J., Lin, C., und Vermette, M., *Conductivity and permeability from microgeometry*, J. Appl. Phys., **56**, 3127-3131, 1984.
17. Nurwidyanto, M. I., Fauzi, U., Handayani, G., *Estimasi permeabilitas untuk pasir lepas dengan ukuran butiran, uji falling head dan analisa citra*, proceedings of HAGI annual meeting, Bandung, Indonesia, 2000.