

# Total Kinetic Energy of Fission Fragments based on Fission Product Data

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( Received: 15 June 2023, Revised: 11 July 2023, Accepted: 31 July 2023 )

## Abstract

Total kinetic energy (TKE) is the physical quantity that must be acquired during a nuclear fission reaction. This energy is used for various purposes, primarily to determine the spectrum of the second proton. This spectrum is advantageous in the design of nuclear reactors. Various techniques for calculating TKE, from microscopic to macroscopic, have been carried out, from statistical to quantum reviews. This whole technique is solely for obtaining TKE accurately. This paper will review the TKE calculation based on the fission products' experimental results. This fission product data can be in the form of raw experimental data or evaluated data. The calculations are carried out within a macroscopic and statistical review framework. The macroscopic view is a liquid drop model, while the statistics use the random number technique. Because the liquid drop model and the random number technique are very standard, this paper does not review them.

*Keywords:* Total Kinetic Energy, Fission Product Data, Macroscopic Framework.

## INTRODUCTION

Fission products can be used as a reference to learn more about the fission event of a nuclide [1]. These physical quantities can be determined through experiments [2,3] or theoretical approaches. Theoretical approach methods can be semi-empirical [4-6], microscopic [7], or macroscopic [8,9]. In addition to this methodology, there are two other types of steps to obtain fission results, namely using potential barriers [10] and direct calculations without going through potential barriers [11]. Although calculations of fission barriers have been carried out for a long time [12-14], there is nothing related to the utilization of the kinetic energy of the magnitude of the distribution of nucleons during the cleavage process. This paper will discuss the calculation of kinetic energy before it becomes a barrier potential curve. This total kinetic energy helps determine the secondary neutron energy spectrum. There have been many reviews regarding this kinetic energy [15-17]. However, people have yet to use fission product data to determine TKE. This work will show how to determine TKE by utilizing fission product data.

Kinetic energy is generated from the combination of the two fission nuclide candidate groups, and this kinetic energy can be viewed from a microscopic point of view for the clustering of many systems of bodies that are reviewed microscopically, such as using the GCM method [18] and fission dynamics with the Langevin approach [19]. Macroscopically it can also be studied through the liquid drop model [20-21]. A macroscopic review of Brosa's fluid drop model [21] looks at the fission process based on three modes: superlong, standard I, and standard II.

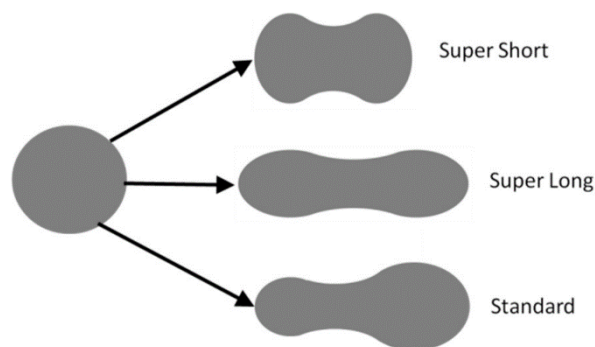


Fig. 1. Illustration of fission modes considered in this work. The neck is the link between two nucleus balls.

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These three modes are the three possible forms of the deformed nuclide. These three probabilities are considered in the combined determination of the nucleon density in the deformed compound nucleus. These modes only exist at low energies or during spontaneous fission events. However, 30 years later, it is shown that these modes can also occur at high energies events ( $\approx 20$  MeV) [22]. It had even been experimentally proven by Balabekyan et al., [23] nine years earlier.

Based on Brosa's model, the TKE calculation is built based on the total mass of fissionable nuclides. The magnitude of this total mass is calculated through the density of nucleons present. This paper will present a TKE calculation following the liquid drop model with the basic shape proposed by Lawrence. This shape is elaborated by three fission modes following the calculation technique from RNRM[24]. Meanwhile, the density is generated by random numbers [25].

## THE METHOD

Total Kinetic Energy is calculated through the formula:

$$TKE = \frac{M_{eff}}{2} \left( \frac{\xi_{max}}{Y} \right)^2 \quad (1)$$

Where  $M_{eff}$  is the effective mass,  $\xi_{max}$  is the initial distance between the two nuclides, and  $Y$  is the time required to separate the two nuclides or reach the distance  $\xi_{max}$ .

$$J(A, \xi, Y) = \frac{\hbar}{2A_i} \int \left( \Psi^*(q: A, \xi, Y) \left( \frac{\partial}{\partial \xi} \Psi(q: A, \xi, Y) \right) - \Psi(q: A, \xi, Y) \left( \frac{\partial}{\partial \xi} \Psi^*(q: A, \xi, Y) \right) \right) dq \quad (6)$$

$q$  is the coordinate of each nucleon.  $A$  is the mass number of the nuclide  $\xi$  is a parameter related to the shape of the nucleus and  $\Psi$  is wave function from slater determinant.

$$M_{eff} = \frac{M_L \cdot M_R}{M_L + M_R} \quad (2)$$

$M_L$  and  $M_R$  nuclide masses of the left and right sides. The quantity  $\xi_{max}$  is obtained by solving the following equation,

$$Y_L = \int_0^{\xi_{max}} \rho(A, \xi, Y) d\xi \quad (3)$$

$$Y_R = \int_0^{-\xi_{max}} \rho(A, \xi, Y) d\xi. \quad (4)$$

$Y_L$  and  $Y_R$  in equations 3 and 4 are the fission yield obtained through experimental data or evaluated data [24], while the nucleon density  $\rho(A, \xi, Y)$  is obtained through the continuity equation,

$$\rho(A, \xi, Y) = \int \left( -\frac{\partial}{\partial \xi} J(A, \xi, Y) \right) dt \quad (5)$$

with  $J(A, \xi, Y)$  is the current nucleon density which is determined by modeling the utilization of random number generation [25].

## RESULTS AND DISCUSSION

The TKE calculations have been carried out following the methodology described above. The results of these calculations can be seen in the following figure.

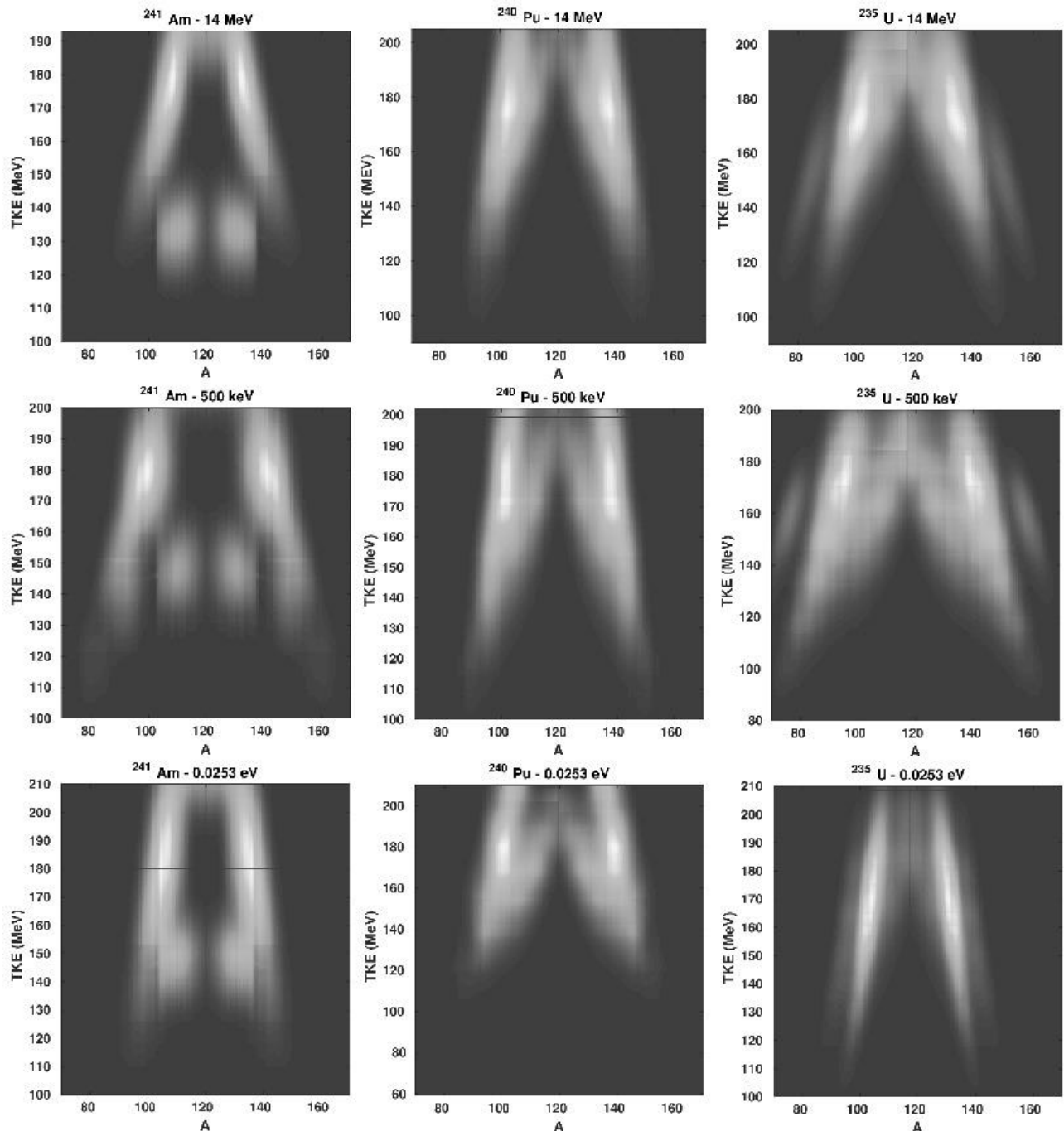


Fig. 2. Total Kinetic Energy for  $^{235}\text{U}$ ,  $^{240}\text{Pu}$  and  $^{241}\text{Am}$  results in various energies.

As the neutron energy increases, the portion of the superlong also increases. ST-1, as the results of this calculation, show a decrease in the portion; this behavior is because of the more significant the incoming neutron energy, the greater the number of fission products close to symmetrical fission.

Figure 2 shows the distribution of TKE based on the mass number A of the fission products. In general, this TKE will follow the pattern of its fission products, especially in the form of fission yield data. For a symmetrical physical product, the TKE value

will dominate at the symmetrical A value. Whereas for asymmetric fission products, TKE will be dominated in the area of A values that often occur or are frequently produced.

For the same nuclide, the distribution of TKE values tends to have the same pattern even though the energies of the incoming neutrons are very different. Thus, the resulting TKE value can be said to be free from the influence of the incoming neutron energy. At low neutron energies such as 0.0253 MeV, TKE tends to have a narrower spread of values. This

Figure shows that the variation in the TKE value is a little for each fission product.

Uranium 235 has a TKE predominance in the energy range of 160 to 200 MeV. Nuclides from fission products with an energy of that range have an extensive range of A values; this differs from  $^{240}\text{Pu}$ , where the A value, which has the most significant TKE energy, is in a small range. This data provides crucial information for their use in reactor design.

## CONCLUSION

Through the calculations that have been done, it can be concluded that the calculation of TKE can be done using fission product data. As with the TKE obtained from other techniques, the proposed technique provides TKE values with a similar pattern to the data pattern from the fission products it refers to. The type of nuclide that will fission dramatically affects the shape of the distribution of the resulting TKE. For  $^{235}\text{U}$ , the TKE values are spread over each fission product, whereas for  $^{241}\text{Am}$ , the TKE values in the symmetrical region are very few in the number of fission products with this TKE value.

## ACKNOWLEDGMENT

Thanks are given to Prof. Anto Sulaksono for help and knowledge sharing.

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