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Alpha Decay Half-Lives Calculation of Even-Even Nuclei in the$62 \leq Z \leq 100$Region using Woods-Saxon Potential

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Abstract

Alpha decay properties of even-even nuclei in the 62 ≤ Z ≤ 100 region are investigated within the Unified Fission Model based on a Modified Woods-Saxon potential. The computed alpha decay half-lives are compared to experimental data and are found to be in good agreement with it. The acquired pattern of the variation of alpha decay half-lives as a function of neutron number are explained by the nuclear shell effect. The experimental alpha decay energy Q is found to have an inverse relation with the alpha decay half-lives. This work has shown that the Unified Fission Model based on a Modified Woods-Saxon potential is sufficient to obtain the values of alpha decay half-lives.

Keywords: Alpha decay, Decay energy, Half-lives, Shell effect, Woods-Saxon.

INTRODUCTION

Alpha decay is a nuclear decay process where a parent nucleus emits an alpha particle. The study of alpha decay functions as a useful tool for the understanding of nuclear information such as nuclear structure and nuclear mass[1]. Alpha decay was discovered by Rutherford in 1899 and it is explained by Gamow [2] and Gurney and Condon [3]as a quantum tunneling phenomenon in the 1920s. In the Unified Fission Model[4], the alpha particle is formed within the nucleus and then the alpha particle penetrates the potential barrier with a probability that depends mainly on the potential barrier. The potential barrier consists of three potentials: the Coulomb potential, the centrifugal potential, and the nuclear interaction. The Coulomb potential and the centrifugal potential are well known. The nuclear potential remains an open problem [5]. In a previous work, a modified Woods-Saxon potential is considered as the nuclear part of the potential barrier for the calculation of alpha decay half-lives of nuclei in the range $120 \leq Z \leq 130$[6].

The purpose of this work is to study the alpha decay properties of nuclei in the range $62 \leq Z \leq 100$within the framework of Unified Fission Modelin which a modified Woods-Saxon is taken as the nuclear part of the potential barrier.The calculated results are then compared to the available experimental results.

THEORETICAL METHOD

The potential barrier is taken as the sum of the Coulomb potential, the centrifugal potential, and the nuclear potential

$$V(r) = V_N(r) + V_c(r) + V_l \,, \quad (1)$$

The nuclear potential is taken as the Modified Woods-Saxon potential

$$V_N(r) = \frac{V_0}{1+\exp\left(\frac{r-R_0}{a}\right)} \,, \quad (2)$$

where

$$R_0 = R_\alpha + R_d - 1.37 \,, \quad (3)$$

and the nuclear charge radii is

$$R_i = 1.27 A_i^{1/3} \,, \quad (4)$$

where $i = \alpha, d$ which is the alpha particle and daughter nucleus.

V_0 and a represent the potential depth and diffuseness written as

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$$V_0 = -44.16[1 - 0.40(I_d + I_\alpha)]\frac{A_d^{\frac{1}{3}}A_\alpha^{\frac{1}{3}}}{A_d^{\frac{1}{3}}+A_\alpha^{\frac{1}{3}}}, \quad (5)$$

$$a = 0.50 + 0.33I_d, \quad (6)$$

with

$$I_i = \frac{N_i - Z_i}{A_i}, \quad (7)$$

where $i = \alpha, d$.

The Coulomb potential between the alpha particle and the daughter nucleus is

$$V_c(r) = \frac{Z_\alpha Z_d e^2}{r}, \quad (8)$$

whereZ_α andZ_d are the proton number of the alpha particle and daughter nucleus, and $e^2 = 1.44\ MeV \cdot fm$[7].

The centrifugal potential is written as

$$V_l(r) = \frac{\hbar^2 l(l+1)}{2\mu r^2}, \quad (9)$$

where $l$is the angular momentum of the alpha particle. In general, only trasitions of ground state to ground state ($l = 0$) occur in alpha decay of even-even nuclei, so $V_l(r) = 0$[8].

According to Gamow's theory of alpha decay, the penetration probability of the alpha particle tunneling through the potential barrier can be calculated as

$$P = \exp\left[-\frac{2}{\hbar}\int_{R_{in}}^{R_{out}}\sqrt{2\mu(V(r) - Q)}dr\right], \quad (10)$$

μ refers to the reduced mass of the alpha-daughter system written as

$$\mu = m\frac{A_\alpha A_d}{A_\alpha + A_d}, \quad (11)$$

where m is the nucleon mass.

R_{in} and R_{out} are the incoming and outgoing points acquired from the equation

$$V(R_{in}) = V(R_{out}) = Q, \quad (12)$$

where Q is the experimental alpha decay energy in MeV.

The half-life of alpha decay is written as

$$T_{1/2} = \frac{\ln 2}{\upsilon P}, \quad (13)$$

where $\upsilon = 2E_\upsilon/h$ represents the amount of collisions against the barrier per second. The empirical vibration energy, E_υ, for alpha decay is

$$E_\upsilon = 0.095Q. \quad (14)$$

We know that[9]

$$\frac{h\ln 2}{2} = 1.433 \cdot 10^{-21}\ MeV \cdot s, \quad (15)$$

and

$$\frac{\hbar^2}{m} = 41.47\ MeV \cdot fm^2, \quad (16)$$

where m is the nucleon mass.

RESULTS AND DISCUSSION

The half-lives of alpha decay is calculated numerically using a Matlab program for each element and its isotopes in the range $62 \le Z \le 100$. The isotopes used for the calculation of alpha decay half-lives have even Z and N numbers, which means they are even-even nuclei. The results are tabulated in Table 1. The first column contains the parent nuclide. The next column has the neutron number (N). The third and fourth columns have the mass number (Ad) and proton number (Zd) of the daughter nuclei. The fifth column contains the experimental decay energy (Q) in MeV [10]. The sixth and seventh columns have the logarithmic values of experimental [11] and calculated half-lives of alpha decay in seconds.

The parent nuclides are split for the sake of explanation and clarity into the following ranges, (i) $62 \le Z \le 72$, (ii) $74 \le Z \le 82$, (iii) $84 \le Z \le 90$, and (iv) $92 \le Z \le 100$.

Fig. 1 and Fig. 3 shows the experimental decay energy Q versus the neutron number N for the even-even isotopes of Sm, Gd, Dy, Er, Yb, Hf and W, Os, Pt, Hg, Pb respectively.It is shown in these two figures that the decay energy Q for each isotope consistently decreases as the neutron number N increases.

Fig. 2 and Fig. 4 shows the experimental and calculated logarithmic half-lives for the even-even nuclei ($62 \le Z \le 72$ for Fig. 2 and $74 \le Z \le 82$ for Fig. 4) versus the neutron number N. It can be seen in both figures that the experimental and calculated half-life values follow the same pattern.

As we examine the four figures, it can be noted that the experimental alpha decay values Q are inversely proportional to its respective half-life values.This is in accordance with Eq. 10 and Eq. 13. A higher value of decay energy leads to a greater probability of the alpha particles being emitted, which leads to a shorter half-life of the parent nuclei. A shorter half-life value indicates lesser stability.

Fig. 5 shows the experimental decay energy Q versus the neutron number N for even-even isotopes of Po, Rn, Ra, and Th. It is shown that the decay energy Q of each isotope decreases over the increase of neutron number N until N=124. A slight increase of decay energy Q is observed at N=126, thena sharp increase of Q value is observed for N=128. After that, the Q value proceeds to decrease as the neutron number N increases.

Fig. 6 are the plots of experimental and calculated logarithmic half-lives of the same isotopes ($84 \le Z \le 90$) versus the neutron number N. It can be seen that the experimental and calculated values of the half-lives follow the same pattern. As the neutron number increases, the half-life of each isotope increases gradually until N=124. A slight drop is observed at N=126, and then a drastic drop is observed in N=128. After that, the half-lives continue to increase as the neutron number increases.

The drop of half-life value at N=128 can be explained by the presence of two neutrons outside the shell closure at N=126. The number 126 is a well-known neutron magic number. Isotopes that contain a magic number for its nucleon number, be it proton or neutron number, have closed shells within its nucleus and are therefore more stable than the next higher number. Hence the drop of half-life value immediately after N=126. After that, as the next shell takes form, the half-lives increase as the neutron number increases.

Fig. 7 shows the experimental decay energy Q versus the neutron number N for even-even isotopes of U, Pu, Cm, Cf, and Fm. Fig. 8 shows the experimental and calculated logarithmic half-lives of the same isotopes versus the neutron number N. It can be seen in these two figures that the decay energy Q has an inverse relation with the half-lives. In Fig. 8, we observe a dip at N=128 for Th isotopes, which has been explained by the shell closure effect.

After the dip, the half-life values of other Th isotopes increase. The other nuclei in this range have increasing values of half-lives, as well. The half-lives of each isotope reach a peak at N=152, after which a drop is observed at N=154. Cf and Fm show an increase after this dip. This may be caused by the shell effect at N=152, which may be another neutron magic number.

CONCLUSION

Alpha decay half-lives for even-even nuclei in the range $62 \leq Z \leq 100$ are investigated within a Unified Fission Model in which the alpha decay process is based on the quantum tunneling mechanism. The potential barrier is taken as the sum of the Coulomb potential, the centrifugal potential, and nuclear potential by considering the latter as a modified Woods-Saxon potential. The alpha decay energy Q is extracted from experimental data. The calculated half-lives are compared to the available experimental values and are found to be in close agreement between each other. The variation of alpha decay half-lives with respect to the neutron number N has been explained based on the shell closure effect and neutron magic number. The experimental alpha decay energy Q is found to have an inverse relation to the alpha decay half-lives. In this work, we have successfully shown that the Unified Fission Model with a modified Woods-Saxon potential is sufficient to obtain the values of alpha decay half-lives for a wide range of isotopes.

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REFERENCES

[1] L'Annunziata MF. Alpha Radiation. Radioactivity 2016:123–34. https://doi.org/10.1016/b978-0-444-63489-4.00004-6.

[2] Gamow G. Zur Quantentheorie des Atomkernes. Zeitschrift Für Phys 1928;51:204–12. https://doi.org/10.1007/BF01343196.

[3] Gurney RW, Condon EU. Wave Mechanics and Radioactive Disintegration. Nature 1928;122:439. https://doi.org/10.1038/122439a0.

[4] Hussein EMA. Chapter One - Mechanisms. In: Hussein EMABT-RM, editor. Radiat. Mech., Oxford: Elsevier Science Ltd; 2007, p. 1–65. https://doi.org/10.1016/B978-008045053-7/50002-1.

[5] Dutt I. The role of various parameters used in proximity potential in heavy-ion fusion reactions: New extension. Pramana - J Phys 2011;76:921–31. https://doi.org/10.1007/s12043-011-0094-3.

[6] Saidi F, Oudih MR, Fellah M, Allal NH. Theoretical study of alpha decay based on Woods-Saxon potential. AIP Conf Proc 2018;1994. https://doi.org/10.1063/1.5048148.

[7] Zhang GL, Le XY. Calculation of α decay half-lives for superheavy elements using the double folding model. Chinese Phys C 2009;33:354–8. https://doi.org/10.1088/1674-1137/33/5/007.

[8] Guo S, Bao X, Gao Y, Li J, Zhang H. The nuclear deformation and the preformation factor in the α-decay of heavy and superheavy nuclei. Nucl Phys A 2015;934:110–20. https://doi.org/https://doi.org/10.1016/j.nuclphysa.2014.12.001.

[9] Poenaru D, Gherghescu R, Greiner W. Alpha- cluster- and fission decay of superheavy nuclei. Rom J Phys 2013;58:1157–66.

[10] Huang WJ, Wang M, Audi G. The AME 2020 atomic mass evaluation. Chinese Phys C 2021;45. https://doi.org/10.1088/1674-1137/abddaf.

[11] Sivasankaran BR, Christas MA, Aarthi K V.,

Larny MJ. Alpha decay half-lives calculation of even-even nuclei in the range $62 \leq Z \leq 118$ using Gamow's theory. Nucl Phys A 2019;989:246–56. https://doi.org/10.1016/j.nuclphysa.2019.06.006.

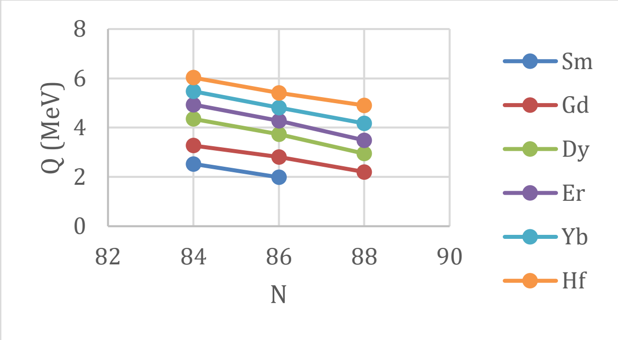

Fig. 1. The experimental alpha decay energies Q for 62 ≤ Z ≤ 72

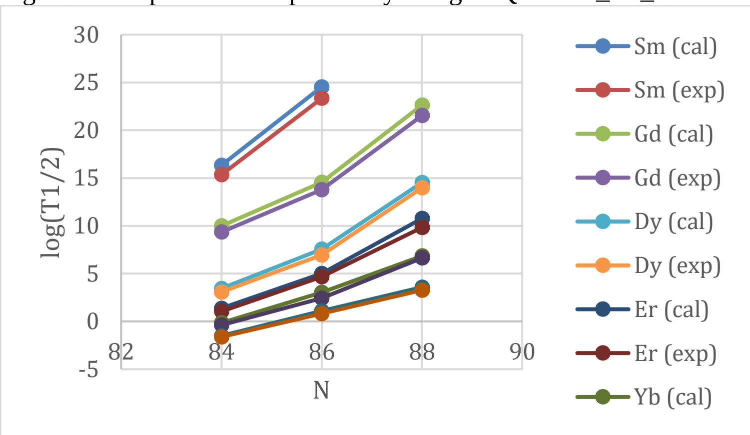

Fig. 2 The experimental and calculated half-lives for 62 ≤ Z ≤ 72

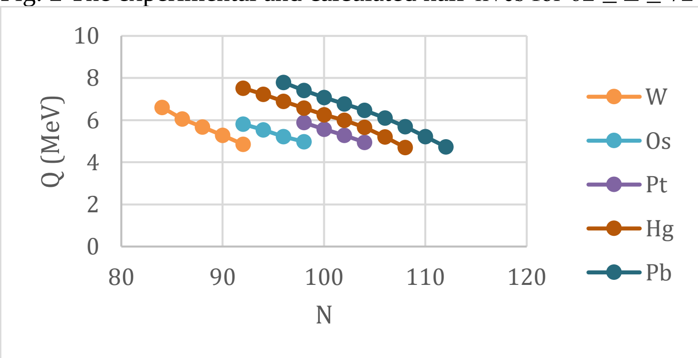

Fig. 3 The experimental alpha decay energies Q for 74 ≤ Z ≤ 82

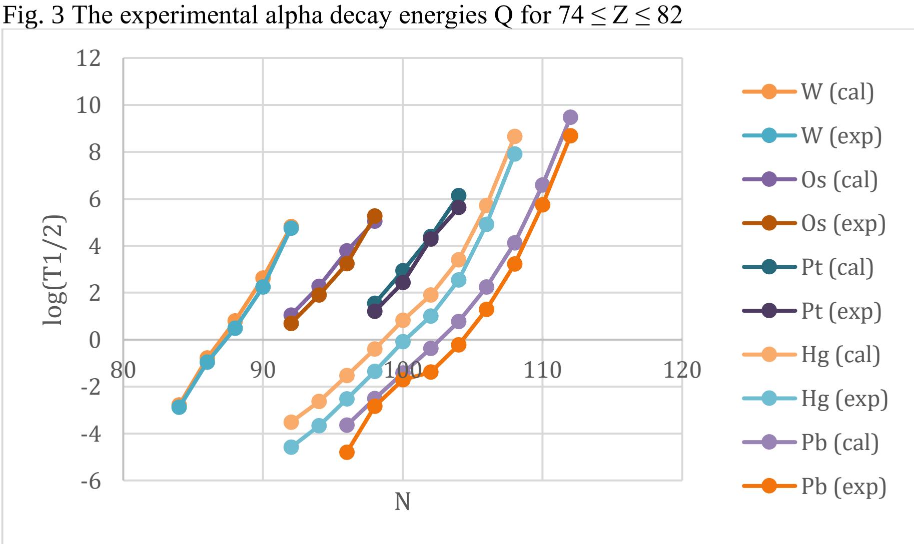

Fig. 4The experimental and calculated half-lives for74 ≤ Z ≤ 82

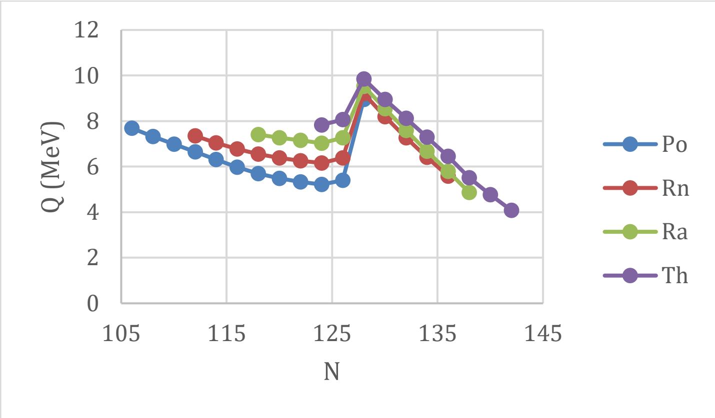

Fig. 5The experimental alpha decay energies Q for 84 ≤ Z ≤ 90

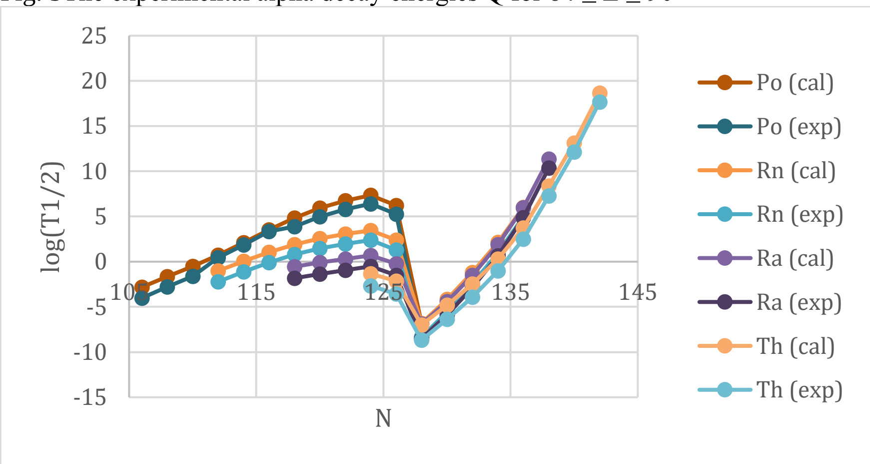

Fig. 6The experimental and calculated half-lives for84 ≤ Z ≤ 90

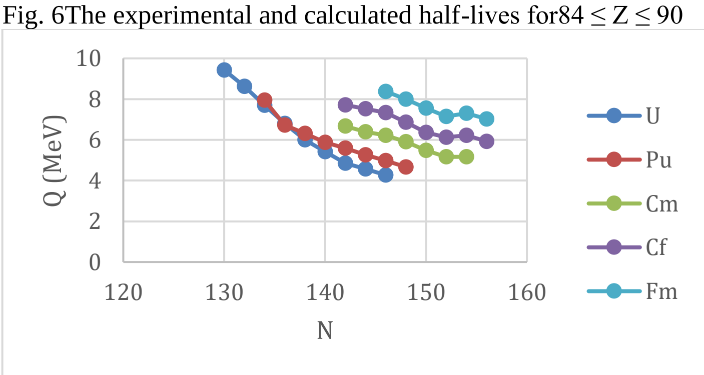

Fig. 7The experimental alpha decay energies Q for 92 ≤ Z ≤ 100

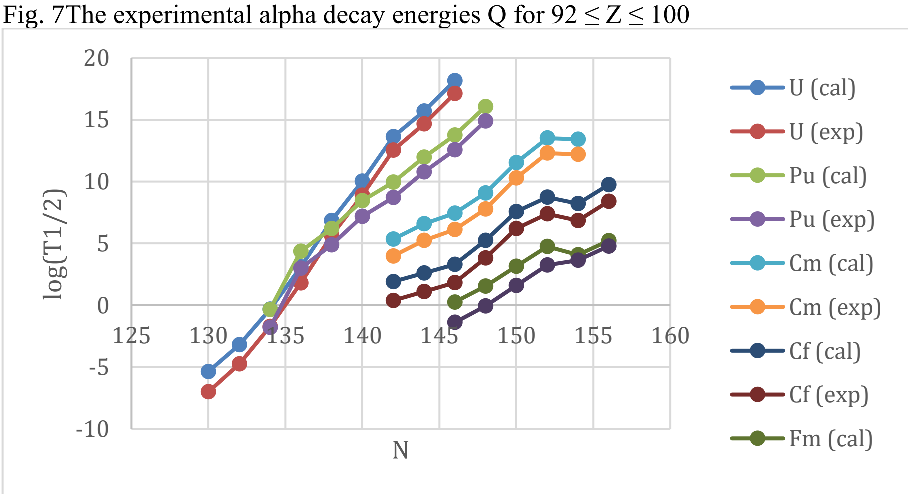

Fig. 8The experimental and calculated half-livesfor 92 ≤ Z ≤ 100

Table 1. The experimental and calculated logarithmic half-lives

Parent	N	Ad	Zd	Q	logT1/2 exp	logT1/2 cal
146-Sm	84	142	60	2.528	15.342	16.334
148-Sm	86	144	60	1.986	23.344	24.534
148-Gd	84	144	62	3.271	9.342	9.987
150-Gd	86	146	62	2.808	13.784	14.552
152-Gd	88	148	62	2.203	21.531	22.633
150-Dy	84	146	64	4.351	3.079	3.444
152-Dy	86	148	64	3.726	6.934	7.565
154-Dy	88	150	64	2.946	13.978	14.522
152-Er	84	148	66	4.934	1.041	1.367
154-Er	86	150	66	4.28	4.681	4.994
156-Er	88	152	66	3.487	9.826	10.762
154-Yb	84	150	68	5.474	-0.387	-0.159
156-Yb	86	152	68	4.811	2.415	3.033
158-Yb	88	154	68	4.172	6.633	6.837
156-Hf	84	152	70	6.028	-1.62	-1.512
158-Hf	86	154	70	5.405	0.806	1.100
160-Hf	88	156	70	4.902	3.279	3.573
158-W	84	154	72	6.613	-2.886	-2.786
160-W	86	156	72	6.065	-0.959	-0.786
162-W	88	158	72	5.677	0.477	0.798
164-W	90	160	72	5.279	2.23	2.616
166-W	92	162	72	4.856	4.74	4.808
168-Os	92	164	74	5.816	0.69	1.044
170-Os	94	166	74	5.537	1.892	2.268
172-Os	96	168	74	5.224	3.23	3.772
174-Os	98	170	74	4.9793	5.255	5.042
176-Pt	98	172	76	5.885	1.204	1.541
178-Pt	100	174	76	5.573	2.431	2.937
180-Pt	102	176	76	5.276	4.279	4.386
182-Pt	104	178	76	4.951	5.623	6.133
172-Hg	92	168	78	7.525	-4.5926	-3.519
174-Hg	94	170	78	7.233	-3.6736	-2.635
176-Hg	96	172	78	6.897	-2.5304	-1.532
178-Hg	98	174	78	6.578	-1.3634	-0.403
180-Hg	100	176	78	6.258	-0.0991	0.825
182-Hg	102	178	78	5.997	0.999	1.894
184-Hg	104	180	78	5.662	2.5411	3.398
186-Hg	106	182	78	5.205	4.9104	5.715
188-Hg	108	184	78	4.705	7.9029	8.650
178-Pb	96	174	80	7.79	-4.801	-3.651

Table 1 (continued)

Parent	N	Ad	Zd	Q	logT1/2 exp	logT1/2 cal
180-Pb	98	176	80	7.415	-2.8505	-2.522
182-Pb	100	178	80	7.076	-1.7094	-1.423
184-Pb	102	180	80	6.774	-1.3872	-0.374
186-Pb	104	182	80	6.47	-0.231	0.763
188-Pb	106	184	80	6.109	1.2896	2.242
190-Pb	108	186	80	5.697	3.2135	4.118
192-Pb	110	188	80	5.221	5.7309	6.581
194-Pb	112	190	80	4.738	8.6729	9.469
190-Po	106	186	82	7.693	-4.0205	-2.821
192-Po	108	188	82	7.32	-2.7956	-1.642
194-Po	110	190	82	6.987	-1.6222	-0.508
196-Po	112	192	82	6.657	0.4281	0.707
198-Po	114	194	82	6.309	1.8635	2.101
200-Po	116	196	82	5.981	3.3309	3.530
202-Po	118	198	82	5.701	3.8764	4.846
204-Po	120	200	82	5.485	4.975	5.924
206-Po	122	202	82	5.327	5.8082	6.744
208-Po	124	204	82	5.215	6.4076	7.337
210-Po	126	206	82	5.407	5.2461	6.206
212-Po	128	208	82	8.954	-8.3588	-6.877
198-Rn	112	194	84	7.349	-2.215	-1.016
200-Rn	114	196	84	7.044	-1.1274	0.036
202-Rn	116	198	84	6.773	-0.1029	1.031
204-Rn	118	200	84	6.546	0.797	1.908
206-Rn	120	202	84	6.384	1.4557	2.552
208-Rn	122	204	84	6.261	1.9609	3.048
210-Rn	124	206	84	6.159	2.3835	3.464
212-Rn	126	208	84	6.385	1.2883	2.406
214-Rn	128	210	84	9.208	-8.3539	-6.813
216-Rn	130	212	84	8.197	-5.5474	-4.154
218-Rn	132	214	84	7.263	-2.4454	-1.182
220-Rn	134	216	84	6.405	0.9874	2.137
222-Rn	136	218	84	5.59	4.9619	6.011
206-Ra	118	202	86	7.415	-1.8259	-0.570
208-Ra	120	204	86	7.273	-1.3525	-0.108
210-Ra	122	206	86	7.152	-0.9445	0.291
212-Ra	124	208	86	7.032	-0.5282	0.698
214-Ra	126	210	86	7.273	-1.5161	-0.249
216-Ra	128	212	86	9.526	-8.5113	-6.900
218-Ra	130	214	86	8.546	-5.8832	-4.415

Table 1 (continued)

Parent	**N**	**Ad**	**Zd**	**Q**	**logT1/2 exp**	**logT1/2 cal**
220-Ra	132	216	86	7.592	-2.8432	-1.508
222-Ra	134	218	86	6.679	0.6633	1.878
224-Ra	136	220	86	5.789	4.8601	5.964
226-Ra	138	222	86	4.871	10.357	11.355
214-Th	124	210	88	7.827	-2.6778	-1.315
216-Th	126	212	88	8.072	-3.5614	-2.156
218-Th	128	214	88	9.849	-8.674	-6.993
220-Th	130	216	88	8.953	-6.3647	-4.813
222-Th	132	218	88	8.127	-3.9028	-2.465
224-Th	134	220	88	7.298	-1.0147	0.312
226-Th	136	222	88	6.451	2.5038	3.723
228-Th	138	224	88	5.52	7.2875	8.395
230-Th	140	226	88	4.77	12.1192	13.145
232-Th	142	228	88	4.082	17.6823	18.641
222-U	130	218	90	9.43	-7.0046	-5.359
224-U	132	220	90	8.62	-4.7366	-3.203
226-U	134	222	90	7.701	-1.7254	-0.316
228-U	136	224	90	6.803	1.7959	3.090
230-U	138	226	90	5.993	5.6324	6.829
232-U	140	228	90	5.414	8.879	10.012
234-U	142	230	90	4.858	12.5337	13.610
236-U	144	232	90	4.573	14.6388	15.690
238-U	146	234	90	4.27	17.1127	18.139
228-Pu	134	224	92	7.94	-1.8101	-0.341
230-Pu	136	226	92	6.716	2.9961	4.360
232-Pu	138	228	92	6.31	4.8802	6.199
234-Pu	140	230	92	5.867	7.1659	8.437
236-Pu	142	232	92	5.593	8.697	9.941
238-Pu	144	234	92	5.256	10.7591	11.971
240-Pu	146	236	92	4.985	12.5607	13.748
242-Pu	148	238	92	4.666	14.8937	16.053
238-Cm	142	234	94	6.67	3.9793	5.336
240-Cm	144	236	94	6.398	5.2462	6.575
242-Cm	146	238	94	6.216	6.125	7.439
244-Cm	148	240	94	5.902	7.7784	9.061
246-Cm	150	242	94	5.475	10.2761	11.515
248-Cm	152	244	94	5.162	12.2909	13.501
250-Cm	154	246	94	5.169	12.193	13.411
240-Cf	142	236	96	7.711	0.3815	1.903
242-Cf	144	238	96	7.517	1.0934	2.596

Table 1 (continued)

Parent	N	Ad	Zd	Q	logT1/2 exp	logT1/2 cal
244-Cf	146	240	96	7.329	1.8108	3.296
246-Cf	148	242	96	6.862	3.8036	5.236
248-Cf	150	244	96	6.361	6.1907	7.567
250-Cf	152	246	96	6.128	7.3744	8.729
252-Cf	154	248	96	6.217	6.8451	8.218
254-Cf	156	250	96	5.927	8.3974	9.742
246-Fm	146	242	98	8.377	-1.3979	0.256
248-Fm	148	244	98	7.994	-0.068	1.541
250-Fm	150	246	98	7.557	1.5831	3.142
252-Fm	152	248	98	7.153	3.2425	4.756
254-Fm	154	250	98	7.308	3.623	4.060
256-Fm	156	252	98	7.027	4.778	5.217
230-Th	140	226	88	4.77	12.1192	13.145
232-Th	142	228	88	4.082	17.6823	18.641
222-U	130	218	90	9.43	-7.0046	-5.359
224-U	132	220	90	8.62	-4.7366	-3.203
226-U	134	222	90	7.701	-1.7254	-0.316
228-U	136	224	90	6.803	1.7959	3.090
230-U	138	226	90	5.993	5.6324	6.829
232-U	140	228	90	5.414	8.879	10.012
234-U	142	230	90	4.858	12.5337	13.610
236-U	144	232	90	4.573	14.6388	15.690
238-U	146	234	90	4.27	17.1127	18.139
228-Pu	134	224	92	7.94	-1.8101	-0.341
230-Pu	136	226	92	6.716	2.9961	4.360
232-Pu	138	228	92	6.31	4.8802	6.199
234-Pu	140	230	92	5.867	7.1659	8.437
236-Pu	142	232	92	5.593	8.697	9.941
238-Pu	144	234	92	5.256	10.7591	11.971
240-Pu	146	236	92	4.985	12.5607	13.748
242-Pu	148	238	92	4.666	14.8937	16.053
238-Cm	142	234	94	6.67	3.9793	5.336
240-Cm	144	236	94	6.398	5.2462	6.575
242-Cm	146	238	94	6.216	6.125	7.439
244-Cm	148	240	94	5.902	7.7784	9.061
246-Cm	150	242	94	5.475	10.2761	11.515
248-Cm	152	244	94	5.162	12.2909	13.501
250-Cm	154	246	94	5.169	12.193	13.411
240-Cf	142	236	96	7.711	0.3815	1.903
242-Cf	144	238	96	7.517	1.0934	2.596

Table 1 (continued)

Parent	**N**	**Ad**	**Zd**	**Q**	**logT1/2 exp**	**logT1/2 cal**
244-Cf	146	240	96	7.329	1.8108	3.296
246-Cf	148	242	96	6.862	3.8036	5.236
248-Cf	150	244	96	6.361	6.1907	7.567
250-Cf	152	246	96	6.128	7.3744	8.729
252-Cf	154	248	96	6.217	6.8451	8.218
254-Cf	156	250	96	5.927	8.3974	9.742
246-Fm	146	242	98	8.377	-1.3979	0.256
248-Fm	148	244	98	7.994	-0.068	1.541
250-Fm	150	246	98	7.557	1.5831	3.142
252-Fm	152	248	98	7.153	3.2425	4.756
254-Fm	154	250	98	7.308	3.623	4.060
256-Fm	156	252	98	7.027	4.778	5.217