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An Interacting Dark Energy Model with Nonminimal Derivative Coupling in the Parameterized Post-Friedmannian Framework

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Abstract

We investigation the parameterization of the cosmological model with the nonminimal derivative coupling of a scalar field where gravity is coupled nonminimally with the derivatives of dark energy components in the form of a scalar field. We follow the parameterized post-Friedmannian approach for the interacting dark energy theories. We show how the big number of free functions can be reduced by limiting certain assumptions to a few non-zero coefficients. We only consider the case that the dark sector contains at most second order in time derivatives of the metric and scalar fields. In this paper, we demonstrate their use through an example of the dark sector interactions model and classify them according to the current literature.

Keywords: cosmological model, nonminimal derivative coupling, scalar field, parameterized post-Friedmannian, interacting dark energy.

INTRODUCTION

The results of observational data such as Cosmic Microwave Background (CMB), Supernova Ia, and Baryon Acoustic Oscillation (BAO) Surveys show that most of the universe components consist of approximately 68% in the form of dark energy, cold dark matter (CDM) around 25%, and baryon ±5% [1,2]. The two dark sectors are the universe's main component and are currently in the accelerated expansion phase. Interaction between dark energy and dark matter does not violate current observational constraints [3]. Interacting dark energy has been studied in the theory of gravitational braneworlds with Lorentz violation [4–6].

The models of dark energy can be done by modified gravity or modified matter [7]. Modified gravity is modifying the left-hand side of Einstein's equation, by adding the new fields to Einstein's equation (fields of scalar, vector, tensor, or the combination) or by adding the extra dimensions. Modified matter is modifying the right-hand side of Einstein's equation, for example, models of quintessence, phantom, k-essence, coupled dark energy, and the chameleon scalar field. The dark energy model with nonminimal derivative coupling (NMDC) between scalar fields and curvature can also give us the universe expansion that accelerated.

At present, various proposals for a modified theory of gravity are still very possible. In addition, these theories must be compared with cosmological

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data such as the parameterized post-Newtonian (PPN) that needs to be confronted with the constraints of the solar system measurements to being understandable and able to solve the problem of dark energy. In the PPN framework, the theory of gravity is constrained simultaneously using test data around the lunar laser and early satellite experiments [8]. The PPN formalism still has weaknesses, i.e. cannot be applied on the cosmological scale, therefore we need to find another formalism to make the parameterization usable for the cosmological theory of modified gravity in general [9]. A new formalism namely the parameterized post-Friedmannian (PPF) was introduced to test the concept of cosmological gravity modification. The PPF framework does not rely on the theory of modified gravity, it is considers several ways to constrain Einstein's linear equations to maintain the properties of the relevant theory of gravity. The PPF framework tests modified gravity as an independent model and then paves the way to classify and test the IDE's theory. It has been done to the theory of modified gravity including the interacting dark energy (IDE) models [10,11].

In this paper, we adopt the PPF formalism with nonminimal derivative coupling (NMDC) of scalar field as in the previous work [12]. We apply the interacting term between the dark sector to reduce the coincidence problem [13]. By looking at the PPF coefficient resulting from linear perturbation of scalar mode in type 1 models from article [14], the physical meaning and characteristic of the model can be expressed.

BACKGROUND AND PERTURBATION EQUATION OF IDE MODEL

The gravitational field equation for the theory of IDE can be written as

$$G_{\mu\nu} = 8\pi G\left(T_{\mu\nu}^{(SM)} + T_{\mu\nu}^{(GDM)} + T_{\mu\nu}^{(DE)}\right) \tag{1}$$

where $G_{\mu\nu}$ is Einstein's tensor, $T_{\mu\nu}^{SM}$ is the energy-momentum tensor of the standard model (SM), $T_{\mu\nu}^{GDM}$ is the energy-momentum tensor of generalized dark matter (GDM), and $T_{\mu\nu}^{DE}$ is the energy-momentum tensor of dark energy (DE). The field equation (1) must fullfill Bianchi's identity $\nabla_\mu G^\mu_\nu = 0$, so that

$$\nabla_\mu\left(T^{(SM)\mu}{}_\nu + T^{(GDM)\mu}{}_\nu + T^{(DE)\mu}{}_\nu\right) = 0 . \tag{2}$$

The standard model particle is considered to be not coupled into the dark sector explicitly so that $\nabla_\mu T^{(SM)\mu}{}_\nu = 0$ and $\nabla_\mu\left(T^{(GDM)\mu}{}_\nu + T^{(DE)\mu}{}_\nu\right) = 0$. The condition causes a coupling current to occur which represents the exchange of energy and momentum between the components of dark energy and dark matter,

$$\nabla_\mu T^{(GDM)\mu}{}_\nu = J_\nu = -\nabla_\mu T^{(DE)\mu}{}_\nu . \tag{3}$$

This coupling current takes the form of metric potential (and its derivatives) and scalar mode which is part of the energy-momentum tensor of the dark sector component.

The cosmological model that will be used in the PPF framework is a model that Friedmann-Robertson-Walker (FRW) model as a cosmological background. The FRW model corresponds to an expanding universe model with a metric containing a time-dependent function. The background spacetime form of this metric is

$$ds^2 = a^2\left(-d\tau^2 + \delta_{ij}dx^i dx^j\right) . \tag{4}$$

where *i*, *j* is the three-dimensional space coordinates index, $d\tau = dt/a$ is the conformal time, and δ_{ij} is the spatial metric. The scale factor as *a* depends on the cosmic time coordinates *t*. We assume a flat spacetime regarding the observation from Wilkinson Microwave Anisotropy Probe (WMAP) [15].

In PPF formalism, the coupling current is parameterized to produce a field equation containing at most a second-order time derivative. In addition, the dark sector component must satisfy two first-order linear field equations in the FRW background resulting from Eq. (3). Initially we reviewed the background FRW spacetime to find an equation that describes the dark sector. Furthermore, the linear perturbation of this background spacetime is reviewed to see its effect on parameterization. In this paper, the background variables will be written with an overbar sign '$\overline{\ldots}$' (unless the scale factor a is always a background variable) and all perturbation tensors will be preceded by a delta sign δ. For example, the momentum energy tensor is separated into two parts, i.e. $T_{\mu\nu} = \overline{T}_{\mu\nu} + \delta\overline{T}_{\mu\nu}$, where the first term is the background part and the second term represents the perturbation part.

The non-zero component of the energy-momentum tensor $\overline{T}_{\mu\nu}$ from the FRW metric a $\overline{\rho} = -\overline{T}_0^0$ as the energy density and $\overline{T}_j^i = \overline{P}\delta_j^i$ as the pressure. The Einstein equation (1) from the metric (4) can be written as

$$3\mathrm{H}^2 = 8\pi G\left(\bar{\rho}_{SM} + \bar{\rho}_{GDM} + \bar{\rho}_{DE}\right) \quad (5)$$

and

$$\mathrm{H}^2 - 2\frac{\ddot{a}}{a} = 8\pi G\left(\bar{P}_{SM} + \bar{P}_{GDM} + \bar{P}_{DE}\right), \quad (6)$$

where $\mathrm{H} = a'/a$ is the conformal Hubble parameter and sign prime '$\ldots'$' shows derivative with respect to conformal time τ.

The non-zero component of the coupling current is

$$\bar{J}_0 \equiv Q \quad (7)$$

where $Q(\tau)$ is the background coupling function. From eq. (4), the continuity equation can be obtained as follows

$$\dot{\bar{\rho}}_I + 3\mathrm{H}\bar{\rho}(1+w_I) = s_I Q \quad (8)$$

where index I represent the mix of components (standard model, generalized dark matter, and dark energy), $w_I \equiv \bar{P}_I/\bar{\rho}_I$ denote the equation of state for each component-*I*, and indicates a constant that is 1 for dark energy, 0 for the standard model, and -1 for generalized dark matter.

The general linear perturbation metric is

$$ds^2 = -a^2(1+2\Psi)d\tau^2 - 2a^2\nabla_i\zeta dtdx^i + a^2\left[\left(1+\tfrac{1}{3}h\right)\gamma_{ij} + D_{ij}\nu\right]dx^i dx^j \quad (9)$$

where Ψ, ζ, h, ν are four scalar mode that is the function of time and space, and $D_{ij} = \nabla_i\nabla_j - \frac{1}{3}\gamma_{ij}\nabla^2$ is the derivative operators that project of longitudinal, traceless, and spatial parts of perturbation. From the metric eq. (9), we have the geometric quantities such as the perturbation of Ricci tensor are

$$\delta R_{00} = \partial_i\Psi^i - \tfrac{1}{2}\ddot{h} - \partial^i\dot{\zeta}_i + H\left(3\dot{\Psi} - \tfrac{1}{2}\dot{h}\right) - \partial^i\zeta_i H, \quad (10)$$

$$\delta R_{0i} = -\dot{\mathrm{H}}\zeta_i + \tfrac{1}{2}\left(\partial_j\partial^j\zeta_i - \partial_i\partial^j\zeta_j\right) - \tfrac{1}{3}\partial_i\dot{h} + \tfrac{1}{2}\partial_j\dot{\nu}_i^j - 2\mathrm{H}^2\zeta_i + 2\mathrm{H}\Psi_i, \quad (11)$$

$$\delta R_{i0} = -\dot{\mathrm{H}}\zeta_i + \tfrac{1}{2}\left(\partial_j\partial^j\zeta_i - \partial_j\partial_i\zeta^j\right) - \tfrac{1}{3}\partial_i\dot{h} + \tfrac{1}{2}\partial^j\dot{\nu}_{ij} - 2\mathrm{H}^2\zeta_i + 2\mathrm{H}\Psi_i, \quad (12)$$

and

$$\begin{aligned}\delta R_{ij} = {} & \dot{\mathrm{H}}\left[\left(-2\Psi + \tfrac{1}{3}h\right)\delta_{ij} + \nu_{ij}\right] \\ & + \mathrm{H}\left[\left(-\dot{\Psi} + \tfrac{5}{6}\dot{h}\right)\delta_{ij} + \dot{\nu}_{ij}\right] \\ & + \tfrac{1}{2}\left(\partial_j\dot{\zeta}_i + \partial_i\dot{\zeta}_j\right) - \tfrac{1}{6}\partial_i\partial_j h \\ & + \left(\tfrac{1}{6}\ddot{h}\delta_{ij} + \tfrac{1}{2}\ddot{\nu}_{ij}\right) + \mathrm{H}\partial_k\zeta^k\delta_{ij} \\ & - \tfrac{1}{6}\partial_k\partial^k h\delta_{ij} + \tfrac{1}{2}\partial_k\partial_i\nu_j^k - \tfrac{1}{2}\partial_k\partial^k\nu_{ij} \\ & - \partial_j\Psi_i - \tfrac{1}{2}\partial_j\partial_i\nu_k^k + \tfrac{1}{2}\partial_j\partial^k\nu_{ik} \\ & + 2\mathrm{H}^2\left[\left(-2\Psi + \tfrac{1}{3}h\right)\delta_{ij} + \nu_{ij}\right] \\ & + \mathrm{H}\left(\partial_j\zeta_i + \partial_i\zeta_j\right) + \tfrac{1}{2}\dot{\nu}_k^k\mathrm{H}\delta_{ij}. \end{aligned} \quad (13)$$

The perturbation of Ricci scalar is

$$\begin{aligned}\delta R = a^{-2}\Big[& -12\Psi\dot{\mathrm{H}} - 2\partial_i\Psi^i + \ddot{h} + 2\partial^i\dot{\zeta}_i \\ & + 3\mathrm{H}\left(-2\dot{\Psi} + \dot{h}\right) - 12\Psi\mathrm{H}^2 \\ & + 6\partial^i\zeta_i\mathrm{H} - \tfrac{2}{3}\partial_i\partial^i h + \tfrac{1}{2}\partial_j\partial^i\nu_i^j\Big] \end{aligned} \quad (14)$$

and the perturbation of Einstein's tensor are

$$\delta G_{00} = \mathrm{H}\left(\dot{h} + \dot{\nu}_i^i\right) + 2\mathrm{H}\partial^i\zeta_i - \tfrac{1}{3}\partial_i\partial^i h + \tfrac{1}{2}\partial_k\partial^i\nu_i^k, \quad (15)$$

$$\delta G_{0i} = 2\dot{\mathrm{H}}\zeta_i + \tfrac{1}{2}\left(\partial_j\partial^j\zeta_i - \partial_i\partial^j\zeta_j\right) + \mathrm{H}^2\zeta_i + 2\mathrm{H}\Psi_i - \tfrac{1}{3}\partial_i\dot{h} + \tfrac{1}{2}\partial_j\dot{\nu}_i^j, \quad (16)$$

$$\delta G_{i0} = 2\dot{\mathrm{H}}\zeta_i + \tfrac{1}{2}\left(\partial_j\partial^j\zeta_i - \partial_j\partial_i\zeta^j\right) + \mathrm{H}^2\zeta_i + 2\mathrm{H}\Psi_i - \tfrac{1}{3}\partial_i\dot{h} + \tfrac{1}{2}\partial^j\dot{\nu}_{ij}, \quad (17)$$

and

$$\begin{aligned}\delta G_{ij} = {} & \dot{\mathrm{H}}\left[\left(4\Psi - \tfrac{2}{3}h\right)\delta_{ij} - 2\nu_{ij}\right] \\ & + \mathrm{H}\left[\left(2\dot{\Psi} - \tfrac{2}{3}\dot{h}\right)\delta_{ij} + \dot{\nu}_{ij}\right] \\ & + \tfrac{1}{2}\left(\partial_j\dot{\zeta}_i + \partial_i\dot{\zeta}_j\right) - 2\mathrm{H}\partial_k\zeta^k\delta_{ij} \\ & - \tfrac{1}{6}\partial_i\partial_j h + \tfrac{1}{6}\partial_k\partial^k h\delta_{ij} - \partial_j\Psi_i \\ & + \tfrac{1}{2}\partial_k\partial_i\nu_j^k - \tfrac{1}{2}\partial_k\partial^k\nu_{ij} + \tfrac{1}{2}\partial_j\partial^k\nu_{ik} \\ & + \mathrm{H}^2\left[\left(2\Psi - \tfrac{1}{3}h\right)\delta_{ij} - \nu_{ij}\right] \\ & + \mathrm{H}\left(\partial_j\zeta_i + \partial_i\zeta_j\right) + \left(-\tfrac{1}{3}\ddot{h}\delta_{ij} + \tfrac{1}{2}\ddot{\nu}_{ij}\right) \\ & + \partial_k\Psi^k\delta_{ij} - \partial^k\dot{\zeta}_k\delta_{ij} - \tfrac{1}{2}\partial_l\partial^k\nu_k^l\delta_{ij}, \end{aligned} \quad (18)$$

where $\nu_{ij} = D_{ij}\nu$ and $\zeta_i = \nabla_i\zeta$. The perturbation of energy-momentum tensor for fluid are

$$T_0^0 = -\bar{\rho}(1+\delta) \quad , \quad (19)$$

$$T_0^i = -\left(\bar{\rho} + \bar{P}\right)\vec{\nabla}^i\theta, \quad (20)$$

$$T_0^i = -\left(\bar{\rho} + \bar{P}\right)\vec{\nabla}^i\theta, \quad (21)$$

and

$$T_j^i = \bar{\rho}(w + \Pi)\delta_j^i + \left(\bar{\rho} + \bar{P}\right)D_j^i\Sigma \quad (22)$$

where δ is density ($\delta \equiv \delta\rho/\bar{\rho}$), θ is the scalar mode of momentum ($u_i = a\nabla_i\theta$), $\prod$ is the dimensionless pressure perturbation ($\prod = \delta P/\bar{\rho}$), and Σ is scalar mode from shear ($\Sigma_{ij} = D_{ij}\Sigma$). The perturbation of Einstein's equations are

$$H\left(\dot{h} + 2\partial^i\zeta_i\right) - 6H^2\Psi - \tfrac{1}{3}\partial_i\partial^i h + \tfrac{1}{2}\partial_k\partial^i v_i^k = 8\pi G a^2 \sum_I \bar{\rho}_I \delta_I \,, \tag{23}$$

$$-2H\Psi^i + \tfrac{1}{3}\partial^i\dot{h} - \tfrac{1}{2}\dot{v}_k^i - \tfrac{1}{2}\left(\partial_k\partial^k\zeta^i - \partial_k\partial^i\zeta^k\right) - 2\dot{H}\zeta^i + 2H^2\zeta^i = 8\pi G a^2 \sum_I \left(\bar{\rho}_I + \bar{P}_I\right)\nabla^i\theta_I \,, \tag{24}$$

$$-2H\Psi_i + \tfrac{1}{3}\partial_i\dot{h} - \tfrac{1}{2}\partial_j\dot{v}_i^j - \tfrac{1}{2}\left(\partial_j\partial^j\zeta_i - \partial_i\partial^j\zeta_j\right) = 8\pi G a^2 \sum_I \left(\bar{\rho}_I + \bar{P}_I\right)\nabla_i\left(\theta_I - \zeta_I\right), \tag{25}$$

and

$$\begin{aligned}&\left\{-\tfrac{1}{3}\ddot{h} + \partial_k\partial^k\left(\tfrac{1}{6}h + \Psi\right) + H\left(2\dot{\Psi} - \tfrac{2}{3}\dot{h}\right) - \tfrac{1}{2}\partial_k\partial^l v_l^k \right.\\ &\left. + \left(4\dot{H} + 2H^2\right)\Psi - \partial^k\dot{\zeta}_k - 2H\partial_k\zeta^k\right\}\delta_j^i \\ &+ \partial^i\partial_j\left(-\tfrac{1}{6}h - \Psi\right) + \tfrac{1}{2}\left(\partial_j\dot{\zeta}^i + \partial^i\dot{\zeta}_j\right) \\ &+ H\left(\partial_j\zeta^i + \partial^i\zeta_j\right) + \tfrac{1}{2}\ddot{v}_j^i - \tfrac{1}{2}\partial_k\partial^k v_j^i \\ &+ \tfrac{1}{2}\partial_k\partial^i v_j^k + \tfrac{1}{2}\partial_j\partial^k v_k^i + H\dot{v}_j^i \\ &= 8\pi G a^2 \sum_I \left[\bar{\rho}_I \Pi_I \delta_j^i + \left(\bar{\rho}_I + \bar{P}_I\right) D_j^i \Sigma\right]\end{aligned} \tag{26}$$

THE PPF FORMALISM

We define two scalar modes of perturbation *q* and *S*,

$$q \equiv \delta J_0 \qquad \text{and} \qquad \vec{\nabla}_i S \equiv \delta J_i . \tag{25}$$

The scalar modes *q* and *S* are parameterized as a linear combination from variables of metric, fluid, and gauge. There are 12 variables for each *q* and *S*. Next, the gauge transformation is done with two constraint equations for each *q* and *S*, it can reduce the number of perturbation variables up to 10 for *q* and *S* respectively. The results are

$$\begin{aligned} q = &\frac{1}{2}\left(\dot{Q} - \mathrm{H}Q\right)V + Q\Psi - 6A_1\hat{\Phi} - 6A_2\hat{\Gamma} + A_3\hat{\delta}_{DE} \\ &+ A_4\hat{\delta}_{GDM} + A_5\hat{\theta}_{DE} + A_6\hat{\theta}_{GDM} + A_7\hat{\Pi}_{DE} \\ &+ A_8\hat{\Pi}_{GDM} + A_9\Sigma_{DE} + A_{10}\Sigma_{GDM}\end{aligned} \tag{28}$$

and

$$\begin{aligned} S = &\frac{1}{2}QV - 6B_1\hat{\Phi} - 6B_2\hat{\Gamma} + B_3\hat{\delta}_{DE} + B_4\hat{\delta}_{GDM} \\ &+ B_5\hat{\theta}_{DE} + B_6\hat{\theta}_{GDM} + B_7\hat{\Pi}_{DE} + B_8\hat{\Pi}_{GDM} \\ &+ B_9\Sigma_{DE} + B_{10}\Sigma_{GDM}\end{aligned} \tag{29}$$

For special case, CDM including is an ideal fluid, so that we get $w_{GDM} = \Pi_{GDM} = \Sigma_{GDM} = 0$. Hereafter, dark energy is assumed does not have a shear $\Sigma_{DE} = 0$ and then the equation of state $w_{DE} = w$. The perturbation of pressure in this case, can be shown in terms of δ_{DE} and θ_{DE}, such as

$$\Pi_{DE} = c_s^2\delta_{DE} + \left(c_s^2 - c_a^2\right)\left[3\left(1+w\right)\mathrm{H} - \frac{Q}{\bar{\rho}_{DE}}\right]\theta_{DE} + \mu\left(\theta_C - \theta_{DE}\right) \tag{30}$$

where c_s^2 are the effective sound speed and c_a^2 is the adiabatic sound speed. The adiabatic sound speed can be shown in the equation of state w as

$$c_a^2 = w + \frac{\dot{w}}{\dfrac{Q}{\bar{\rho}_{DE}} - 3\mathrm{H}\left(1+w\right)} . \tag{31}$$

It can be determined that A_7 and B_7 are zero based on the description above.

Additionally, we can observe the conformal Newtonian gauge condition with $\zeta = \nu = 0$ and $V = 0$. Then, the equation *q* and *S* from eq. (28) and (29) becomes

$$\begin{aligned} q = &Q\Psi - 6A_1\Phi - 6A_2\left(\dot{\Phi} + \mathrm{H}\Psi\right) + A_3\delta_{DE} \\ &+ A_4\delta_C + A_5\theta_{DE} + A_6\theta_C \end{aligned} \tag{32}$$

and

$$\begin{aligned} S = &-6B_1\Phi - 6B_2\left(\dot{\Phi} + \mathrm{H}\Psi\right) + B_3\delta_{DE} + B_4\delta_C \\ &+ B_5\theta_{DE} + B_6\theta_C \end{aligned} \tag{33}$$

where A_i and B_i ($i \in 1\ldots6$) are the unknown function.

SPECIFIC MODEL OF NMDC

Generally, the scalar fields in the cosmological model can appear in three forms of Lagrangian, i.e. minimal coupling, nonminimal coupling, and nonminimal derivative coupling (NMDC). Inflation will be absent if the presence of NMDC does not allow attractor inflation [16]. It has been reviewed in the context of late-time acceleration and inflation. To restore the cosmological constant, we use the coupling constants of NMDC with behaviors of the de Sitter universe [17].

The generalized NMDC to the five-dimensional Universal Extra Dimension (UED) model has an accelerated expansion of the universe. The cosmological constant can be written in the form of a combination of the coupling constant of this model. The effect of NMDC on changes in the value of gravitational constant G has also been carried out in [18,19]. An expansion of the NMDC model has also been carried out on the braneworld model of Randall-Sundrum in five-dimensional [20].

The action of NMDC model in [13], that is

$$S=\int d^4x\sqrt{-g}\left[\frac{R}{2}-\frac{1}{2}\left(\in g_{\mu\nu}-\eta G_{\mu\nu}\right)\partial^{\mu}\phi\partial^{\nu}\phi -V(\phi)-F(\phi)L_m\right]. \quad (34)$$

where $\in$ denotes the quintessence field by taking the value +1 and -1 for the phantom field, $V(\phi)$ is the scalar field potential, $F(\phi)=\beta e^{\alpha\phi}$ indicates the interacting term between the dark sector with constants α and β > 0, and L_m represent the Lagrangian density of matter (except baryons and radiation which are subdominant and supposed to be minimally coupled to gravity). In this work, we assume $8\pi G=c=\hbar=1$. The action variations related to the metric tensor to get the Einstein's field equation, as follows

$$G_{\mu\nu}=\in T_{\mu\nu}{}^{(\phi)}+\eta T_{\mu\nu}{}^{(\eta)}+T_{\mu\nu}{}^{(m)}-g_{\mu\nu}V(\phi), \quad (35)$$

with

$$\begin{aligned}
T_{\mu\nu}{}^{(\phi)} &= \nabla_{\mu}\phi\nabla_{\nu}\phi-\frac{1}{2}g_{\mu\nu}\nabla_{\alpha}\phi\nabla^{\alpha}\phi \\
T_{\mu\nu}{}^{(\eta)} &= \frac{1}{2}\nabla_{\mu}\phi\nabla_{\nu}\phi R-\nabla_{\alpha}\phi\nabla_{\mu}\phi R^{\alpha}_{\nu}-\nabla_{\alpha}\phi\nabla_{\nu}\phi R^{\alpha}_{\mu} \\
&\quad -\nabla^{\alpha}\phi\nabla^{\beta}\phi R_{\mu\alpha\nu\beta}-\nabla_{\mu}\nabla^{\alpha}\phi\nabla_{\nu}\nabla_{\alpha}\phi \\
&\quad +\nabla_{\mu}\nabla_{\nu}\phi\nabla_{\alpha}\nabla^{\alpha}\phi+\frac{1}{2}\left(\nabla_{\alpha}\phi\nabla^{\alpha}\phi\right)^2 G_{\mu\nu} \\
&\quad -g_{\mu\nu}\left[\frac{1}{2}\nabla^{\alpha}\nabla^{\beta}\phi\nabla_{\alpha}\nabla_{\beta}\phi-\frac{1}{2}\left(\nabla_{\alpha}\nabla^{\alpha}\phi\right)^2 \right. \\
&\qquad \left. -\nabla_{\alpha}\phi\nabla_{\beta}\phi R^{\alpha\beta}\right] \quad (36)
\end{aligned}$$

where $T_{\mu\nu}{}^{(\phi)}$ and $T_{\mu\nu}{}^{(\eta)}$ related to the variation that depend on the scalar field and $T_{\mu\nu}{}^{(m)}$ is the matter energy-momentum tensor.

In order for the third derivative terms of scalar fields can be eliminated, we can take the relationship between two couplings constants like $2\xi+\eta=0$. The resulting still have a complicated form so it is very difficult to find a solution directly. To find constraints that can make the equation simpler, we need to consider certain cases. In this paper, the De Sitter solution provides constraints in the form of parameter values for the Hubble constant and a scalar field shown in a linear function with respect to time (constant) as in [7]. Therefore, the background values of density and pressure from the NMDC action (34) are

$$\rho_{\phi}=\frac{\dot{\bar{\phi}}^2}{2a^2}+\eta a^{-4}\left(-3\dot{H}+\frac{17}{2}H^2\right)\dot{\bar{\phi}}^2 +V(\phi)+F(\phi)\rho_m \quad (37)$$

and

$$p_{\phi}=\frac{\dot{\bar{\phi}}^2}{2a^2}+\eta a^{-4}\left(\dot{H}-\frac{23}{2}H^2\right)\dot{\bar{\phi}}^2 -V(\phi)+F(\phi)\rho_m\,. \quad (38)$$

The coupling current that will be used in parameterization with this NMDC is the coupling current from model of coupled dark matter to dark energy type 1[14], as follows

$$J_{\mu}=-\beta_{\phi}\rho_C\nabla_{\mu}\phi \quad (39)$$

where β_{ϕ} is free function to scalar field ϕ ($\beta_{\phi}\equiv\frac{\partial\beta}{\partial\phi}$) and subscript C denotes CDM. The function of background coupling is

$$Q=J_0=-\beta_{\phi}\bar{\rho}_C\dot{\bar{\phi}}\,. \quad (40)$$

The equation of parameterization q dan S becomes

$$q=-\beta_{\phi\phi}\bar{\rho}_C\dot{\bar{\phi}}\varphi+Q\left(\delta_C+\frac{\dot{\varphi}}{\dot{\bar{\phi}}}\right)$$

and

$$S=Q\frac{\varphi}{\dot{\bar{\phi}}}\,. \quad (41)$$

We use the equation of state parameter of dark energy $w=-1$ to constrain the model [21]. From the eq. (37-38), we get

$$\dot{\bar{\phi}}^2=-\frac{2a^2F(\phi)\rho_m}{\in+\frac{16}{3}\eta\rho_{DE}}\,. \quad (42)$$

Next, from the equation perturbation of density and pressure (39-40), we find

$$\frac{\dot{\varphi}}{\dot{\bar{\phi}}}=\Psi-\frac{3\left(\in\rho_{DE}+\frac{16}{3}\eta\rho^2{}_{DE}\right)}{2F(\phi)\rho_m\left(3\in-19\eta\rho_{DE}\right)}\delta_{DE} -\frac{V_{\phi}}{\left(3\in-19\eta\rho_{DE}\right)\dot{\bar{\phi}}}\theta_{DE} \quad (43)$$

and

$$\Psi=\frac{64\delta_m-114}{32\delta_m-41} \quad (44)$$

Both above equations are substituted into the equation (41)

$$q = Q\Psi + Q\delta_C - \frac{3\left(\in \rho_{DE} + \frac{16}{3}\eta\rho^2{}_{DE}\right)Q}{2F(\phi)\rho_m\left(3\in -19\eta\rho_{DE}\right)}\delta_{DE} - \left(\frac{V_\phi Q}{\left(3\in -19\eta\rho_{DE}\right)\dot{\bar{\phi}}} - \frac{2a^2F(\phi)\rho_m\beta_{\phi\phi}\bar{\rho}_C}{\in + \frac{16}{3}\eta\rho_{DE}}\right)\theta_{DE} \quad (45)$$

and

$$S = Q\theta_{DE} \quad (46)$$

where $\theta_{DE} = \varphi / \dot{\bar{\phi}}$ is the momentum divergence of scalar field. If

$$\frac{1+8\xi\pi G\left(15\rho + P\right)}{1+16\xi\pi G\rho} = 1, \quad (47)$$

we will have the PPF coefficients if comparing the equations (45) with (32) and (46) with (33), we get

$$A_1 = A_2 = A_6 = 0,$$

$$A_3 = -\frac{3\left(\in \rho_{DE} + \frac{16}{3}\eta\rho^2{}_{DE}\right)Q}{2F(\phi)\rho_m\left(3\in -19\eta\rho_{DE}\right)},$$

$$A_4 = Q,$$

$$A_5 = -\left(\frac{V_\phi Q}{\left(3\in -19\eta\rho_{DE}\right)\dot{\bar{\phi}}} - \frac{2a^2F(\phi)\rho_m\beta_{\phi\phi}\bar{\rho}_C}{\in + \frac{16}{3}\eta\rho_{DE}}\right),$$

and

$$B_5 = Q. \quad (47)$$

CONCLUSION

This work, we parameterization of the interacting dark energy model, i.e. coupled dark matter to dark energy type 1 with NMDC in the PPF formalism. These theories depends on number of parameter, such as the background field energy density, the chosen coupling function, the background coupling, the potential, etc. For type 1 with NMDC, we have 3 values of A whose coefficient is not zero and only 1 non-zero B coefficient. This results corresponding with previous work [12]. In particular, from all the cases we studied, the coeffients A_1, A_6, B_1, B_2, and B_4 were always zero. It would be indeed be very interesting to find models for which any of these coefficients in nonzero.

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