Indonesian Journal of Physics
Vol. 33. No.1, 17 October 2022
Journal homepage: http://ijphysics.fi.itb.ac.id

Gravitational Wave Propagation for The Generalized Proca Theories

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(Received: 28 February 2022, Revised: 30 May 2022, Accepted: 11 October 2022)

Abstract
In general relativity, a gravitational wave propagates with the speed of light, but in the alternative theories of gravity, propagation speed could deviate from the speed of light due to the modification of gravity. Gravitational waves are influenced by modified gravity during propagation at the cosmological distance. In this paper, we investigate the propagation of a gravitational wave of the generalized Proca theories by considering gravitational wave as the gravitational field propagates in spacetime as a wave perturbing flat spacetime. We show that the arbitrary functions G3, G4, and G5 can be the sources of deviation of the speed of the gravitational wave.

Keywords: gravitational waves, modification gravity, Proca theories.

INTRODUCTION

Gravitational waves from a binary black hole had been detected in 2015 and 2016 by a LIGO and Virgo Collaboration [1-4]. In 2017, LIGO and Virgo had detected gravitational waves from a binary neutron star [5,6]. These observation results demonstrated the evidence of gravitational waves. It will convey information on the dynamic of systems in the strong field gravity limit and can be a probe to test theories of gravity. An alternative of theories of gravity that consider a massive vector field is the generalized Proca theories, which is constructed by derivating self-interaction with only three propagating degrees of freedom, two transverse and one longitudinal. The analysis was based on studying the Hessian matrix and assuring the propagation of a second class constraint [7]. It is expected that gravitational waves from the binary enable us to test that gravity theory in strong and dynamical regimes precisely. To treat the test, it is necessary to have a parameterized theory. We use the propagation equation of a gravitational wave which describes the modification of gravity at cosmological scales to parameterize the generalized Proca theories. The parameterized variation of modified gravity will affect gravitational wave propagation. It needs to obtain gravitational wave equation then get the speed of gravitational wave to learn how gravitational wave propagates base on the alternative theories of gravity. We investigate the gravitational wave propagation for the generalized Proca theories. Our paper is organized as follows. In

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section 2 we review the previous study on the generalized Proca theory. Next in section 3, we review the theory of background cosmology that we use. In sec 4, we construct formulation to gain deviation of gravitational wave's speed of the generalized Proca theory at cosmological scales. Finally, we summarize our findings in section 5.

THE GENERALIZED PROCA THEORIES

In generalized Proca theories, the vector field A^{μ} posses two transverse polarizations and one longitudinal scalar mode nonminimally coupled to gravity. The generalized Proca theories describes by the four dimension action [7,8]

$$S = \int d^4x\sqrt{-g}\left(\mathcal{L}_F + \sum_{i=2}^{5}\mathcal{L}_i\right) \tag{1}$$

$$\mathcal{L}_F = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} \tag{2}$$

$$\mathcal{L}_2 = G_2\left(X\right) \tag{3}$$

$$\mathcal{L}_3 = G_3\left(X\right)\nabla_{\mu}A^{\mu} \tag{4}$$

$$\begin{aligned}\mathcal{L}_4 = {} & G_4\left(X\right)R + G_{4X}\left(X\right)\Big[\left(\nabla_{\mu}A^{\mu}\right)^2 \\ & + c_2\nabla_{\rho}A_{\sigma}\nabla^{\rho}A^{\sigma} - \left(1+c_2\right)\nabla_{r}A_{\sigma}\nabla^{\sigma}A^{\rho}\Big]\end{aligned} \tag{5}$$

$$\begin{aligned}\mathcal{L}_5 = {} & G_5\left(X\right)G_{\mu\nu}\nabla^{\mu}A^{\nu} - \frac{1}{6}G_{5X}\left(X\right)\Big[\left(\nabla_{\mu}A^{\mu}\right)^3 \\ & - 3d_2\nabla_{\mu}A^{\mu}\nabla_{\rho}A_{\sigma}\nabla^{\rho}A^{\sigma} \\ & - 3\left(1-d_2\right)\nabla_{\mu}A^{\mu}\nabla_{\rho}A_{\sigma}\nabla^{\sigma}A^{\rho} \\ & + \left(2-3d_2\right)\nabla_{\rho}A_{\sigma}\nabla^{\gamma}A^{\rho}\nabla^{\sigma}A_{\gamma} \\ & + 3d_2\nabla_{\rho}A_{\sigma}\nabla^{\gamma}A^{\rho}\nabla_{\gamma}A^{\sigma}\Big]\end{aligned} \tag{6}$$

Here, A_{μ} is a vector field with $X = -\frac{1}{2}A_{\mu}A^{\mu}$ and $F_{\mu\nu} = \nabla_{\mu}A_{\nu} - \nabla_{\nu}A_{\mu}$. Energi density and pressure of the generalized Proca theory is obtained as

$$\begin{aligned}\rho_M = {} & G_2 - G_{2X}\phi^2 - 3G_{3X}H\phi^3 + 6G_4H^2 \\ & - 6\left(2G_{4X} + G_{4XX}\phi^2\right)H^2\phi^2 \\ & + G_{5XX}H^3\phi^5 + 5G_{5X}H^3\phi^3\end{aligned} \tag{7}$$

$$\begin{aligned}P_M = {} & -G_2 + \dot{\phi}\phi^2G_{3X} - 2G_4\left(3H^2 + 2\dot{H}\right) \\ & + 2G_{4X}\phi\left(3H^2\phi + 2H\dot{\phi} + 2\dot{H}\phi\right) \\ & + 4G_{4XXX}H\dot{\phi}\phi^3 - G_{5XX}H^2\dot{\phi}\phi^4 \\ & + G_{5X}H\phi^2\left(2\dot{H}\phi + 2H^2\phi + 3H\dot{\phi}\right)\end{aligned} \tag{8}$$

Variation of the action with respect to A^{μ} leads to

$$\begin{aligned}\phi\Big(& G_{2X} + 3G_{3X}H\phi + 6G_{4X}H^2 + 6G_{4XX}H^2\phi^2 \\ & - 3G_{5X}H^3\phi - G_{5XX}H^3\phi^3\Big) = 0\end{aligned} \tag{9}$$

COSMOLOGICAL BACKROUND

Considering a perfectly homogeneous and isotropic spatially flat cosmological background in conformal time takes the form of the Friedmann-Robertson-Walker (FRW). We define the flat Friedmann-Lemaitre-Robertson-Walker (FLRW) metric as

$$ds^2 = -dt^2 + a^2\left(t\right)\delta_{ij}dx^i dx^j \tag{10}$$

The background equation of motion in the standard form [9,10]

$$3H^2M_P^2 = a^2\left(\rho_m + \rho_{DE}\right) \tag{11}$$

M_P is the constant Planck mass, ρ_m is the energy density of cold dark matter, and ρ_{DE} is the energy density of dark energy, which is to embrace all modified gravity effects. We could have instead defined M_P with an effective time-dependent mass M_* on the contrary of M_P for a a different ρ_{DE}. ρ_{DE} is an additional degree of freedom that couples to the metric in a non-trivial way. Standard matter component such as baryons and photons have been assumed to be minimally coupled to the metric, therefore the way they contribute to the background is unchanged. Their propagation and evolution are determined by standard geodesic in the given metric background.

Considering a small perturbations around the background of universe is the total metric

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu}\,,\ \left|h\right| \ll \left|\bar{g}\right| \tag{12}$$

Where $\eta_{\mu\nu}$ is the background FRW metric and $h_{\mu\nu}$ is a linear perturbation whose transverse and traceless part $h_{\mu\nu}^{TT}$ encodes the gravitational wave amplitude. The other components of $h_{\mu\nu}$ determine the amplitude of matter energy-density perturbations. We consider modified gravity theories that do not propagate additional tensor modes, therefore $h_{\mu\nu}^{TT}$ carries all the information on the evolution of the metric polarizations.

PROPAGATION GRAVITATIONAL WAVE OF THE GENERALIZED PROCA THEORIES

In general relativity, gravitational waves are the result of perturbation of metric flat as written in equation (13). Gravitational waves carry energy and momentum in traveling as a ripple of curvature. For the generalized Proca theories, Einsten equation can be written as

$$G_{\mu\nu} = \kappa^2 T_{\mu\nu}^{(A)} \tag{13}$$

With $G_{\mu\nu}$ is the Einstein tensor, $T_{\mu\nu}^{(A)}$ is energy-momentum tensor of the generalized Proca theories. Energy-momentum of the generalized Proca theories is obtained by using variaton action of equation (3) to (7)

$$T_{\mu\nu(1)} = \frac{1}{4} g_{\mu\nu} F_{\alpha\beta} F^{\alpha\beta} \tag{14}$$

$$T_{\mu\nu(2)} = g_{\mu\nu} G_2(X) + G_{2X}(X) A_\nu A_\mu \tag{15}$$

$$\begin{aligned} T_{\mu\nu(3)} &= G_3(X) \nabla_\alpha A^\alpha g_{\mu\nu} + G_{3X} A_\nu A_\mu \\ &+ 2G_3(X) g_{\mu\alpha} g_{\nu\beta} \nabla^\alpha A^\beta \end{aligned} \tag{16}$$

$$\begin{aligned} T_{\mu\nu(4)} &= g_{\mu\nu} G_4(X) g^{\alpha\beta} R_{\alpha\beta} + g^{\alpha\beta} R_{\alpha\beta} A_\nu A_\mu \\ &- 2G_4(X) R_{\mu\nu} + g_{\mu\nu} G_{4X} \left(\nabla_\alpha A^\alpha\right)^2 \\ &+ G_{4XX} A_\nu A_\mu \left(\nabla_\alpha A^\alpha\right)^2 + 4G_{4X} g_{\alpha\mu} g_{\beta\nu} \nabla_\alpha A^\alpha \nabla^\beta A^\alpha \\ &+ c_2 g_{\mu\nu} G_{4X} \nabla_\rho A_\sigma \nabla^\rho A^\sigma + c_2 G_{4XX} A_\nu A_\mu \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ &+ 2c_2 G_{4X} g_{\mu\rho} g_{\gamma\nu} \nabla^\gamma A_\sigma \nabla^\rho A^\sigma \\ &- 2c_2 G_{4X} \nabla_\nu A_\sigma \nabla_\mu A^\sigma - (1+c_2) g_{\mu\nu} G_{4X} \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &- (1+c_2) G_{4XX} A_\nu A_\mu \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &- 2(1+c_2) g_{\mu\rho} g_{\nu\alpha} G_{4X} \nabla^\alpha A_\sigma \nabla^\sigma A^\rho \\ &+ 2(1+c_2) G_{4X} \nabla_\rho A_\mu \nabla_\nu A^\rho \end{aligned} \tag{17}$$

$$\begin{aligned} T_{\mu\nu(5)} &= g_{\mu\nu} G_5(X) G_{\lambda\gamma} \nabla^\lambda A^\gamma + G_{5X} A_\nu A_\mu G_{\lambda\gamma} \nabla^\lambda A^\gamma \\ &- G_5(X) g_{\lambda\mu} g_{\gamma\nu} R \nabla^\lambda A^\gamma + G_5(X) g_{\lambda\gamma} R_{\mu\nu} \nabla^\lambda A^\gamma \\ &- 2G_5(X) G_{\mu\nu} \nabla_\nu A^\nu - \frac{1}{6} g_{\mu\nu} G_{5X} \left(\nabla_\alpha A^\alpha\right)^3 \\ &- \frac{1}{6} G_{5XX} A_\nu A_\mu \left(\nabla_\alpha A^\alpha\right)^3 \\ &- G_{5X} g_{\alpha\mu} g_{\beta\nu} \left(\nabla_\mu A^\mu\right)^2 \nabla^\beta A^\alpha \\ &+ \frac{1}{2} d_2 g_{\mu\nu} G_{5X} \nabla_\gamma A^\gamma \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ &+ \frac{1}{2} d_2 G_{5XX} A_\nu A_\mu \nabla_\gamma A^\gamma \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ &+ d_2 G_{5X} g_{\alpha\mu} g_{\beta\nu} \nabla^\beta A^\alpha \nabla_\rho A_\sigma \nabla^\rho A^\sigma \\ &+ d_2 g_{\lambda\mu} g_{\rho\nu} G_{5X} \nabla_\gamma A^\gamma \nabla^\lambda A_\sigma \nabla^\rho A^\sigma \\ &- d_2 G_{5X} \nabla_\gamma A^\gamma \nabla_\nu A_\sigma \nabla_\mu A^\sigma \\ &+ \frac{1}{2}(1-d_2) g_{\mu\nu} G_{5X} \nabla_\alpha A^\alpha \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &+ \frac{1}{2}(1-d_2) G_{5XX} A_\nu A_\mu \nabla_\alpha A^\alpha \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &+ (1-d_2) G_{5X} g_{\alpha\mu} g_{\beta\nu} \nabla^\beta A^\alpha \nabla_\rho A_\sigma \nabla^\sigma A^\rho \\ &+ (1-d_2) G_{5X} \nabla_\alpha A^\alpha g_{\lambda\mu} g_{\rho\nu} \nabla^\lambda A_\sigma \nabla^\sigma A^\rho \\ &- (1-d_2) G_{5X} \nabla_\alpha A^\alpha \nabla_\rho A_\nu \nabla_\mu A^\rho \\ &- \frac{1}{6}(2-3d_2) g_{\mu\nu} G_{5X} \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma \\ &- \frac{1}{6}(2-3d_2) G_{5XX} A_\nu A_\mu \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma \\ &- \frac{1}{3}(2-3d_2) G_{5X} g_{\lambda\mu} g_{\rho\nu} \nabla^\lambda A_\sigma \nabla^\gamma A^\rho \nabla^\sigma A_\gamma \\ &+ \frac{1}{3}(2-3d_2) G_{5X} \nabla_\rho A_\sigma \nabla_\mu A^\rho \nabla^\sigma A_\nu \\ &+ \frac{1}{3}(2-3d_2) G_{5X} \nabla_\rho A_\nu \nabla^\gamma A^\rho \nabla_\mu A_\gamma \\ &- \frac{1}{2} d_2 g_{\mu\nu} G_{5X} \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma \\ &- \frac{1}{2} d_2 G_{5XX} A_\nu A_\mu \nabla_\rho A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma \\ &- d_2 G_{5X} g_{\lambda\mu} g_{\rho\nu} \nabla^\lambda A_\sigma \nabla^\gamma A^\rho \nabla_\gamma A^\sigma \\ &+ d_2 G_{5X} \nabla_\rho A_\sigma \nabla_\mu A^\rho \nabla_\nu A^\sigma \\ &- d_2 G_{5X} \nabla_\rho A_\sigma \nabla^\gamma A^\rho g_{\alpha\mu} g_{\gamma\nu} \nabla^\alpha A^\sigma \end{aligned} \tag{18}$$

We assume the following field configuration for the vector field $A^\mu = (\phi(t), 0, 0, 0)$ and $\phi(t)$ is a vector field depend on time. With Dropping higher order terms of $h_{\mu\nu}$, the Riemann curvature tensor is

$$\begin{aligned} R_{\alpha\mu\beta\nu} &= \frac{1}{2}\left(\partial_\alpha \partial_\nu h_{\mu\beta} + \partial_\mu \partial_\beta h_{\alpha\nu} \right. \\ &\left. - \partial_\alpha \partial_\beta h_{\mu\nu} - \partial_\mu \partial_\nu h_{\alpha\beta}\right) \end{aligned} \tag{19}$$

The Ricci tensor

$$R_{\mu\nu} = \frac{1}{2}\left(\partial_\alpha \partial_\nu h_\mu^\alpha + \partial_\mu \partial_\alpha h_\nu^\alpha - \Box h_{\mu\nu} - \partial_\mu \partial_\nu h\right) \tag{20}$$

Where $\Box = \partial_\mu \partial^\mu$.

On a cosmological background, interactions between the new degree of freedom and curvature or metric can affect the speed of propagation of gravitational wave (c_T) that make the effective Planck mass (M_*) extend in time [12-15]. Propagation equation of gravitational wave is generally is given by [14]

$$h_{ij}'' + (2+\nu) H h_{ij}' + \left(c_T^2 k^2 + a^2 \mu^2\right) h_{ij} = a^2 \Gamma_{\gamma ij} \qquad (21)$$

Where h_{ij} is a tensor perturbation and the prime is a derivative with respect to conformal time, a is the scale factor, H is the Hubble parameter in conformal time, ν is the Planck mass run rate, c_T is the phase velocity of a gravitaional wave and μ is the graviton mass. Γ denotes extra sources generating gravitational wave. In the limit of $c_T = 1$ and $\nu = \mu = \Gamma = 0$, the equation of propagation (21) is reduced to the standard form of the general relativity.

We identified $\nu = \alpha_M$. Here, α_M has been defined as

$$\alpha_M \equiv \frac{d \ln\left(M_*/M_P\right)^2}{d \ln a} = \frac{2}{H} \frac{M_*'}{M_*} \qquad (22)$$

$$\alpha_T \equiv c_T^2 - 1 \qquad (23)$$

We get the deviation of gravitational wave of the generalized Proca theories as

$$\alpha_T = \Big(G_2(X) + G_3(X)\dot{\phi} + 7G_3(X) H\phi + 9G_{4X} H\phi + G_{4X} H^2\phi^2 - 3G_5(X)\dot{\phi}H^2 - 6G_5(X)\phi H\dot{H} - 9G_5(X)\phi H^3 - \frac{41}{6} G_{5X} H^3\phi^3 - 3d_2 G_{5X} H^3 \phi^3 + \frac{1}{6}(1-6d_2) G_{5X}\dot{\phi}^3 + \frac{1}{2}(6+d_2) G_{5X}\dot{\phi} H^2 \phi^2 + \frac{7}{2} G_{5X} H\phi\dot{\phi}^2 \Big) \frac{a^2}{k^2} - 1$$

(24)

Where k is a wavenumber. This result is beyond our expectation because it still contains the variable d. And the role of Plank mass is played by

$$M_*^2 = 2G_4 - 4G_{4X}\phi^2 + \frac{5}{3} G_{5X} H \phi^3 \qquad (25)$$

Planck mass is influenced by the vector field ϕ. This result can be used to parameterize the generalized Proca theories for the future work.

CONCLUSION

In this paper, we consider the linearized Einstein theory in finding the deviation of gravitational wave's speed for the generalized Proca theories. From the result, we can see that all the arbitrary function have influenced the gravitational wave's speed because in the arbitrary function involve Proca mass that as the sources of gravitational waves in the generalized Proca theories. The rate of mass Planck is influenced by arbitrary function G_4 and G_5, because those function is the result interaction with curvature. In the future work we will relate the results of this study with the results of observations to parameterize the generalized Proca theories.

ACKNOWLEDGMENT

The works of M is support in part by Scholarship of LPDP RI. The work of AS and FPZ is supported by PDUPT of The Ministry of Research, Technology, and Higher Education of the Republic of Indonesia. We thank our friends in laboratory of theoretical physics for useful discussion.

REFERENCES

[1] Abbott B.P et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Physical Review Letters* **116.061102**, 2016,.
[2] Abbott B.P et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Physical Review Letters* **116.241103**, 2016.
[3] Abbott B.P et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Physical Review Letters* **118.221101**, 2017.
[4] Abbott B.P et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Physical Review Letters* **119.141101**, 2017.
[5] Abbott B.P et al, (LIGO Scientific Collaboration and Virgo Collaboration), *Physical Review Letters* **119.161101**, 2017.
[6] Abbott B.P et al, (Virgo, Fermi-GBM,INTEGRAL, LIGO Scientific), *Astrophys. J.*848, **L13**, 2017.
[7] Heisenberg L, *Journal od Cosmology and Astroparticle Physics*, JCAP05(2014)**015**, 2014.
[8] De Felice A. Et al, *Journal od Cosmology and Astroparticle Physics*, JCAP06(2016)**048**, 2016.
[9] De Felice, A., L. Heisenberg, and S. Tsujikawa, *Physical Review*, **D95**, 123540(2017) , 2017.
[10] Lagos M et all, *Physical Review* **D99**, 083504(2019) , 2019.
[11] Bellini E and Sawicki I, *Journal od Cosmology and Astroparticle Physics*, JCAP07(2014)**050**, 2014.
[12] Saltas, I.D., et al, *Physical Review Letters*, PRL **113**,191101(2014) , 2014.
[13] Nishizawa A, *Physical Review*, D97, 104037(2018) , 2018.
[14] Arai S, and A. Nishizawa, *Physical Review*, **D97**, 10438(2018), 2018.
[15] Nunes, R.C., M.E.S. Alves, and J.C.N de

Araujo, *Physical Review* **D99**, 0840022(2019) , 2019.
[16] Amendola L., et al, *Journal od Cosmology and Astroparticle Physics*, JCAP08(2018)**030**, 2018.