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Study of Entangled K-meson and Its Decoherence

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Abstract

In this paper, the entangled K-meson model and decoherence phenomenon in the system is studied. Using the Lindblad equation, the dynamical equation of the entangled K-meson system that interacts with the environment is obtained. We find that the non-Hermitian Hamiltonian of the system makes a completely positive and trace-preserving map (CPT-map) on the space of the density matrix does not satisfy trace preserving properties. We also find that purity of density matrix can be less than $1/d$ which does not satisfy the property of purity. From the dynamical equation, parameters related to the decoherence of the system, decoherence parameter (λ) and effective decoherence parameter (ζ), are determined. Using Standard Least-Squares method, we obtain $\zeta = 0.13 \pm 0.865$. This result is in accordance with the results of the references that use the effective variance method, $\zeta = 0.13^{+0.16}_{-0.15}$. We show that $\zeta = 0.13^{+0.16}_{-0.15}$ corresponds with the references' result, $\lambda = (1.84^{+2.50}_{-2.17}) \times 10^{-12}$ MeV. The value of both parameters are close to zero relative to $\zeta = 1$ or $\lambda \to \infty$. It means that the interaction between system and environment does not affect the system significantly. Therefore, quantum properties in the system related to the entanglement of the strangeness are preserved.

Keywords: decoherence, entanglement, k-meson, open quantum system.

INTRODUCTION

Quantum mechanics is a physical theory which allows entanglement phenomenon. In this phenomenon, a state vector called entangled state represents a pair of particles such that each particle cannot be represented as one state [1], [2]. Entanglement has some interesting applications such as quantum teleportation [3], [4], [5], [6], [7], [8], superdense coding [4], [9], [10], and quantum cryptography [4], [11], [12].

However, interaction between a quantum system and its environment can cause quantum effects to disappear. This phenomenon, called decoherence, could be measured quantitatively [7], [13], [14], [15], [16], [17], [18], [19], [20], [21], [22]. On the other hand, open quantum theory, related to quantum transport [23], [24], [25], explains a total system which contains the system of interest and environment [26], [27], [28]. From this explanation, we know a decoherence phenomenon is important to be studied further so that we could utilize the effect from quantum mechanics better.

Among any other type of particles, K-meson is relatively interesting because we can study basics of quantum mechanics, Bell Inequality, and symmetry (T, CP, and CPT) with the help of interference phenomenon in K-meson studies. These studies are feasible because of mass difference value between K_{S} (short-lived-state) and K_{L} (long-lived-state) as well as K_{S} lifetime [16], [20], [21], [29]. Therefore, in this paper, the entangled K-meson system and its decoherence phenomenon are studied.

MODEL OF ENTANGLED K-MESON SYSTEM BASED ON LINDBLAD EQUATION

This section is based on some references [15], [16], [18], [19], [20]. Neutral K-meson is categorized into K^0 and $\overline{K^0}$. K^0 consists of one down quark and one strange antiquark ($d\bar{s}$). Whereas $\overline{K^0}$ consists of down antiquark and one strange quark ($\bar{d}s$). The state of K-meson can be characterized by strangeness. Strangeness operator can be expressed as

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$$S = |K^0\rangle\langle K^0| - |\overline{K^0}\rangle\langle\overline{K^0}|. \quad (1)$$

Suppose P is a parity operator. The fact that K-meson is pseudoscalar (odd parity) means $P|K^0\rangle = -|K^0\rangle$ and $P|\overline{K^0}\rangle = -|\overline{K^0}\rangle$. On the other hand, Suppose C is a charge conjugation operator. The fact that $\overline{K^0}$ is antiparticle of K^0 means $C|K^0\rangle = |\overline{K^0}\rangle$. Thus, we can get $CP|K^0\rangle = -|\overline{K^0}\rangle$ and $CP|\overline{K^0}\rangle = -|K^0\rangle$. From here, we can obtain eigen equations for CP operator, $CP|K_1^0\rangle = +|K_1^0\rangle$ and $CP|K_2^0\rangle = -|K_2^0\rangle$ with $|K_1^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle - |\overline{K^0}\rangle\right)$ and $|K_2^0\rangle = \frac{1}{\sqrt{2}}\left(|K^0\rangle + |\overline{K^0}\rangle\right)$ as the eigenstates.

We know that, for K-meson, CP symmetry violation in weak interaction happens with probability $|\varepsilon| \approx 10^{-3}$ (ε is CP violating parameter). On the other hand, from experimental results, there are two types of K-meson based on its lifetime. K-meson that has a lifetime of $\Gamma_{\mathrm{S}}^{-1} = \tau_{\mathrm{S}} = (0.8954 \pm 0.0004) \times 10^{-10}$ s is K_{S} (short-lived-state). Whereas K-meson that has a lifetime of $\Gamma_{\mathrm{L}}^{-1} = \tau_{\mathrm{L}} = (5.116 \pm 0.021) \times 10^{-8}$ s is K_{L} (long-lived-state). Their mass difference is $\Delta m = m_{\mathrm{L}} - m_{\mathrm{S}} = (3.484 \pm 0.006) \times 10^{-6}$ eV [30]. From here, we know that $|K_{\mathrm{S}}\rangle$ and $|K_{\mathrm{L}}\rangle$ can be constructed from superposition of $|K_1^0\rangle$ and $|K_2^0\rangle$ to $|K_{\mathrm{S}}\rangle = \frac{1}{\sqrt{2}}\left(|K_1^0\rangle + \varepsilon|K_2^0\rangle\right)$ and $|K_{\mathrm{L}}\rangle = \frac{1}{\sqrt{2}}\left(\varepsilon|K_1^0\rangle + |K_2^0\rangle\right)$. By changing bases of these states to $|K^0\rangle$ and $|\overline{K^0}\rangle$ as well as supposing $p = 1 + \varepsilon$, $q = 1 - \varepsilon$, and $N = \sqrt{|p|^2 + |q|^2}$, we get normalized states as follows:

$$|K_{\mathrm{S}}\rangle = \frac{1}{N}\left(p|K^0\rangle - q|\overline{K^0}\rangle\right),$$
$$|K_{\mathrm{L}}\rangle = \frac{1}{N}\left(p|K^0\rangle + q|\overline{K^0}\rangle\right). \quad (2)$$

Hamiltonian for its decay is $H = M - \frac{i}{2}\Gamma$ with M, related to mass, and Γ, a decay-matrix, is a Hermitian operator. Its related eigen equation is $H|K_{\mathrm{S,L}}\rangle = \lambda_{\mathrm{S,L}}|K_{\mathrm{S,L}}\rangle$ with $\lambda_{\mathrm{S,L}} = m_{\mathrm{S,L}} - \frac{i}{2}\Gamma_{\mathrm{S,L}}$. On the other hand, entangled K-meson is expressed as

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}\left(|e_1\rangle - |e_2\rangle\right), \quad (3)$$

which is a Bell States [4], [8], [18], [31], [32], [33], [34], [35], [36] with $|e_1\rangle = |K_{\mathrm{S}}\rangle_l \otimes |K_{\mathrm{L}}\rangle_r$ and $|e_2\rangle = |K_{\mathrm{L}}\rangle_l \otimes |K_{\mathrm{S}}\rangle_r$. Index l is related to a particle that moves to the left side and index r is related to a particle that moves to the right side. Its Hamiltonian is $H = H_l \otimes 1_r + 1_l \otimes H_r$. On the other hand, the CPLEAR experiment [37], whose results will be used in the next section, is not sensitive to the effect of CP violation. Therefore, we can assume CP invariance is not violated ($p = q = 1$). It means $\langle K_{\mathrm{S}}|K_{\mathrm{L}}\rangle = 0 \Rightarrow \langle e_1|e_2\rangle = 0$ and, from equation (3), we obtain

$$\rho(0) = \frac{1}{2}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - |e_1\rangle\langle e_2| - |e_2\rangle\langle e_1|\right). \quad (4)$$

Lindblad equation (Gorini - Kossakowski - Sudarshan - Lindblad equation) is an equation that represents a subsystem dynamics in a Markovian system as the total system. Its evolution operator has to be completely positive and trace-preserving maps (CPT-maps) [26], [28], [38], [39], [40], [41]. We can use it to determine the evolution of the density matrix as follows.

$$\mathcal{L}\rho(t) \equiv \dot{\rho}(t) = -i(H\rho(t) - \rho(t)H^\dagger) + \sum_k \left(L_k\rho(t)L_k^\dagger - \frac{1}{2}\{L_k^\dagger L_k, \rho(t)\}\right), \quad (5)$$

with jump operator or Lindblad operator $L_k = \sqrt{\lambda}P_k$, decoherence parameter λ, projector $P_k = |e_k\rangle\langle e_k|$, $k = 1, 2$, and $\{A, B\} = AB + BA$ (suppose A and B are operators). Operator $\mathcal{L}$ called superoperator, is a linear operator which acts on a density matrix and transforms it to a density matrix in Fock-Liouville Space (FLS) [26], [28], [42], [43], [44].

Suppose $\rho(t) = \sum_{ij}^2 \rho_{i,j}(t)|e_i\rangle\langle e_j|$ and $\Gamma = \frac{1}{2}(\Gamma_{\mathrm{S}} + \Gamma_{\mathrm{L}})$. Using equation (5), we get $\dot{\rho}_{ij}(t) = -2\Gamma\rho_{ij}(t)$ for $i = j$ and $\dot{\rho}_{ij}(t) = -(2\Gamma + \lambda)\rho_{ij}(t)$ for $i \neq j$ which leads to $\rho_{ij}(t) = \rho_{ij}(0)e^{-2\Gamma t}$ for $i = j$ and $\rho_{ij}(t) = \rho_{ij}(0)e^{-(2\Gamma+\lambda)t}$ for $i \neq j$. From equation (4), we know that $\rho_{11}(0) = \rho_{22}(0) = \frac{1}{2}$ and $\rho_{12}(0) = \rho_{21}(0) = -\frac{1}{2}$. Thus,

$$\rho(t) = \frac{1}{2}e^{-2\Gamma t}\left(|e_1\rangle\langle e_1| + |e_2\rangle\langle e_2| - e^{-\lambda t}(|e_1\rangle\langle e_2| + |e_2\rangle\langle e_1|)\right). \quad (6)$$

From equation (6), we know that density matrix represents pure state for $t = 0$ and does not represent pure state for $t > 0$ with $\lambda \neq 0$. The larger the value of t is, the smaller the coherences effect in the density matrix. It means that quantum effects, represented by coherences, become more vanished as t becomes larger due to environmental influence (a decoherence phenomenon happens).

We know that $\dot{\rho}(t) = \mathcal{L}\rho(t) \Rightarrow \rho(t) = e^{\mathcal{L}t}\rho(0)$ which means $e^{\mathcal{L}t}$ transform density matrix $\rho(0)$ to density matrix $\rho(t)$. Thus, $\mathcal{L}$ is generator of CPT represented as $\mathcal{V}(t) = e^{\mathcal{L}t}$ [26], [28]. We should take a note that the result of $\mathrm{Tr}[\rho(0)] = 1$ is different with the result of $\mathrm{Tr}[\mathcal{V}(t)\rho(0)] = \mathrm{Tr}[\rho(t)] = e^{-2\Gamma t}$ even though $\mathcal{V}(t)$ should fulfill trace-preserving properties. It happens because the Hamiltonian of the system is not hermitian. This is in accordance with $\mathrm{Tr}[\dot{\rho}(t)] = -2\Gamma e^{-2\Gamma t} \neq 0$ which means the trace of density matrix is changed or not preserved according to t. On the other hand, the value of trace result of derivation of density matrix, negative, is in accordance with $\mathrm{Tr}[\rho(t)] = e^{-2\Gamma t}$ whose value becomes smaller.

This problem, caused by non-hermitian Hamiltonian in this system, has been solved in other works [45], [46]. We can work with this Hamiltonian by some modifications to the Hilbert space and the dynamical equation. We extend the Hilbert space to become $\mathcal{H}_{\mathrm{tot}} = \mathcal{H} \oplus \mathcal{H}_0$, with $\mathcal{H}_{\mathrm{tot}}$ is total Hilbert space, $\mathcal{H}$ is the Hilbert space of the system, and $\mathcal{H}_0$ is space related to the "decay states". "Decay state" is the result of dissipation which projects system states to it. Using $H = M - \frac{i}{2}\Gamma$, the dynamical equation becomes $\dot{\rho}(t) = -i[M, \rho(t)] - \{\frac{1}{2}\Gamma, \rho(t)\}$. Let $\Gamma = B^\dagger B$, with $B : \mathcal{H} \to \mathcal{H}_0$, we get $\mathrm{Tr}[\dot{\rho}(t)] = -\mathrm{Tr}[B^\dagger B\rho(t)] \neq 0$. We can add $B\rho(t)B$ to the equation to become $\dot{\rho}(t) = -i[M, \rho(t)] -\{\frac{1}{2}\Gamma, \rho(t)\} + B\rho(t)B$ so that $\mathrm{Tr}[\dot{\rho}(t)] = 0$. This means that the trace of the density matrix is preserved [46].

We also get the purity of the density matrix, $\mathrm{Tr}[\rho^2(t)] = \frac{1}{2}e^{-4\Gamma t}(1 + e^{-2\lambda t})$. For $t = 0$, the value of purity is one, which corresponds with $\rho(0) = |\psi^-\rangle\langle\psi^-|$ (pure state). On the other hand, for $t > 0$, its value is less than one (mixed state). It means that evolution of the density matrix transforms a pure state into a mixed state (a decoherence phenomenon occurs). The value of purity also can be less than $\frac{1}{2}$ even though its value should fulfill $\frac{1}{d} \leq \mathrm{Tr}[\rho^2] \leq 1$ (in this system, $d = 2$). It happens because one of the conditions for density matrix, $\mathrm{Tr}[\rho] = 1$, is not fulfilled when $t > 0$. This property of purity is not satisfied when $\mathrm{Tr}[\rho] \neq 1$.

DECOHERENCE PARAMETER RELATED TO ENTANGLED K-MESON SYSTEM

This section is based on some references [15], [16], [18], [19], [20] with Standard Least Square as a fitting method [47], [48]. In the CPLEAR experiment, entangled K-meson are produced and then their strangeness is detected [37]. Consider a case for $\overline{K^0}$ detected at the right side at $t = t_r$ and K^0 detected at the left side at $t = t_l$ ($t_r \leq t_l$). Strangeness measurement at the right side is represented by operator $S_r^+ = 1_l \otimes |K^0\rangle\langle K^0|_r$ and at the left side is represented by operator $S_l^- = |\overline{K^0}\rangle\langle\overline{K^0}|_l$. After measurement at the right side, the density matrix of the K-meson which has not been measured becomes $\rho_l(t = t_r; t_r) = \mathrm{Tr}_r\left(S_r^+\rho(t_r)\right)$. From here, we can get the expected value of this case,

$$\begin{aligned} \mathrm{P}(\overline{K^0}, t_l; K^0, t_r) &= \mathrm{Tr}\left(S_l^- \rho_l(t_l; t_r)\right) \\ &= \mathrm{Tr}\left(\mathrm{Tr}_r\left(S_r^+\rho(t_r)\right)\right). \end{aligned} \tag{7}$$

We use $|K_{\mathrm{S}}\rangle$ and $|K_{\mathrm{L}}\rangle$ as basis to determine equation (7). We obtain

$$\begin{aligned} \rho_l(t = t_r; t_r) = \frac{1}{4}e^{-2\Gamma t_r}\big(&|K_{\mathrm{S}}\rangle\langle K_{\mathrm{S}}|_l + |K_{\mathrm{L}}\rangle\langle K_{\mathrm{L}}|_l \\ &-e^{-\lambda t_r}(|K_{\mathrm{S}}\rangle\langle K_{\mathrm{L}}|_l + |K_{\mathrm{L}}\rangle\langle K_{\mathrm{S}}|_l)\big). \end{aligned} \tag{8}$$

Suppose density matrix related to a particle which moves to the left side is

$$\begin{aligned} \rho_l(t; t_r) = \rho_{\mathrm{SS}}(t; t_r)|K_{\mathrm{S}}\rangle\langle K_{\mathrm{S}}|_l \\ +\rho_{\mathrm{SL}}(t; t_r)|K_{\mathrm{S}}\rangle\langle K_{\mathrm{L}}|_l + \rho_{\mathrm{LS}}(t; t_r)|K_{\mathrm{L}}\rangle\langle K_{\mathrm{S}}|_l \\ +\rho_{\mathrm{LL}}(t; t_r)|K_{\mathrm{L}}\rangle\langle K_{\mathrm{L}}|_l. \end{aligned} \tag{9}$$

This particle is assumed to not experience decoherence phenomenon for $t > t_r$. It means we should use $\dot{\rho}_l(t; t_r) = -i(H\rho_l(t; t_r) - \rho_l(t; t_r)H^\dagger)$ to determine its evolution. We get

$$\begin{aligned} \dot{\rho}_l(t; t_r) = -\Gamma_{\mathrm{S}}\rho_{\mathrm{SS}}(t; t_r)|K_{\mathrm{S}}\rangle\langle K_{\mathrm{S}}|_l \\ +(i\Delta m - \Gamma)\rho_{\mathrm{SL}}(t; t_r)|K_{\mathrm{S}}\rangle\langle K_{\mathrm{L}}|_l \\ -(i\Delta m + \Gamma)\rho_{\mathrm{LS}}(t; t_r)|K_{\mathrm{L}}\rangle\langle K_{\mathrm{S}}|_l \\ -\Gamma_{\mathrm{L}}\rho_{\mathrm{LL}}(t; t_r)|K_{\mathrm{L}}\rangle\langle K_{\mathrm{L}}|_l. \end{aligned} \tag{10}$$

Suppose C_{ij} is constants with $i, j = \mathrm{S}, \mathrm{L}$. Then, we obtain $\rho_{\mathrm{ii}}(t; t_r) = C_{\mathrm{ii}}e^{-\Gamma_{\mathrm{i}}t}$ for $i = j$, $\rho_{\mathrm{SL}}(t; t_r) = C_{\mathrm{SL}}e^{(i\Delta m - \Gamma)t}$, and $\rho_{\mathrm{LS}}(t; t_r) = C_{\mathrm{LS}}e^{-(i\Delta m + \Gamma)t}$. From equation (8) and (9), we know that $\rho_{\mathrm{SS}}(t = t_r; t_r) = \rho_{\mathrm{LL}}(t = t_r; t_r) = \frac{1}{4}e^{-2\Gamma t_r}$ and $\rho_{\mathrm{SL}}(t = t_r; t_r) = \rho_{\mathrm{LS}}(t = t_r; t_r) = -\frac{1}{4}e^{-(2\Gamma + \lambda)t_r}$. Thus, C_{ij} and $\rho_{ij}(t; t_r)$ can be determined. By substituting $t=t_l$, we get $\mathrm{P}(\overline{K^0},t_l;K^0,t_r)$. The expectation value of other cases can be determined in the same way. Suppose $\Delta t=t_l-t_r$, the expectation value for each case are as follows:

$$\mathrm{P}(K^0,t_l;\overline{K^0},t_r)=\mathrm{P}(\overline{K^0},t_l;K^0,t_r)$$
$$=\frac{1}{8}e^{-2\Gamma t_r}(e^{-\Gamma_\mathrm{S}\Delta t}+e^{-\Gamma_\mathrm{L}\Delta t}$$
$$+2e^{-\lambda t_r}\cos(\Delta m\Delta t)e^{-\Gamma\Delta t}),$$

(11)

and

$$\mathrm{P}(K^0,t_l;K^0,t_r)=\mathrm{P}(\overline{K^0},t_l;\overline{K^0},t_r)$$
$$=\frac{1}{8}e^{-2\Gamma t_r}(e^{-\Gamma_\mathrm{S}\Delta t}+e^{-\Gamma_\mathrm{L}\Delta t}$$
$$-2e^{-\lambda t_r}\cos(\Delta m\Delta t)e^{-\Gamma\Delta t}). \quad (12)$$

On the other hand, asymmetry of probability (A) is

$$A(t_l,t_r)=\frac{\mathrm{P}(K^0,t_l;\overline{K^0},t_r)+\mathrm{P}(\overline{K^0},t_l;K^0,t_r)-\left(\mathrm{P}(K^0,t_l;K^0,t_r)+\mathrm{P}(\overline{K^0},t_l;\overline{K^0},t_r)\right)}{\mathrm{P}(K^0,t_l;\overline{K^0},t_r)+\mathrm{P}(\overline{K^0},t_l;K^0,t_r)+\mathrm{P}(K^0,t_l;K^0,t_r)+\mathrm{P}(\overline{K^0},t_l;\overline{K^0},t_r)}. \quad (13)$$

We don't know which particle will be detected first. Thus t_r in the right side of equation (11) and (12) is changed to $\min\{t_l,t_r\}$. Suppose $\Delta\Gamma=\Gamma_\mathrm{L}-\Gamma_\mathrm{S}$, asymmetry of probability is obtained as

$$A_\lambda(t_l,t_r)=\frac{e^{-\lambda\min\{t_l,t_r\}}\cos(\Delta m\Delta t)}{\cosh(\frac{1}{2}\Delta\Gamma\Delta t)}. \quad (14)$$

Suppose $A_\mathrm{QM}(t_l,t_r)$ is the asymmetry of probability for a system that does not experience a decoherence phenomenon and we get $A_\lambda(t_l,t_r)=A_\mathrm{QM}(t_l,t_r)e^{-\lambda\min\{t_l,t_r\}}$. This decoherence model can be related to the phenomenon model with the same system [14], [20]. In the phenomenon model, there is an effective decoherence parameter (ζ) related to decoherence parameter (λ) whose relation is

$$\zeta(t_l,t_r)=1-e^{-\lambda\min\{t_l,t_r\}}. \quad (15)$$

This makes equation (14) can be rewritten as

$$A_\lambda(t_l,t_r)=\left(1-\zeta(t_l,t_r)\right)A_\mathrm{QM}(t_l,t_r). \quad (16)$$

Effective decoherence parameter (ζ) describes how significant the effect from decoherence is. Total decoherence described by $\zeta=0$ and $\zeta=1$ means that there is no decoherence which happens. On the other hand, decoherence parameter (λ) describes how significant the effect from interaction between system and environment is. From equation (15), we know that $\lambda=0$ is corresponding with $\zeta=0$ and $\lambda\to\infty$ is corresponding with $\zeta=1$.

To determine an effective decoherence parameter, we can use the Standard Least-Squares method. As experimental results, we use asymmetry data from CPLEAR. In this experiment, there are two configuration, C(0) and C(5). In C(0), the asymmetries are $A_\lambda(0)=0.81\pm0.17$ and $A_{QM}(0)=0.93$. Whereas, in C(5), the asymmetries are $A_\lambda(5)=0.48\pm0.12$ and $A_{QM}=0.56$ [37]. The result from this method is $\zeta=0.13\pm0.865$. This value is in accordance with the result from other works, $\zeta=0.13^{+0.16}_{-0.15}$ [14], [20], which use the effective variance method [49].

Decoherence parameter (λ) can be determined by knowing the effective decoherence parameter (ζ) and measurement time (t_l and t_r). On the other hand, from other references, we know that $\lambda=(1.84^{+2.50}_{-2.17})\times10^{-12}$ MeV, $\zeta=0.13^{+0.16}_{-0.15}$, and $\min\{t_l,t_r\}=0.55\tau_\mathrm{S}$ [15], [16], [18], [19], [20]. These parameters are corresponding to each other when we substitute their value to equation (15). Using equation (14) and (16), we show that these parameters are corresponding to each other in Figure 2. In the figure, we know that graphs which result from λ almost coincide with graphs which result from ζ. When using equation (14) and (15), we should take a note that the unit of λ is actually s^{-1}. On the other hand, unit of time can be represented as $\hbar/$eV. So, the unit of λ actually can be represented as eV$/\hbar$ with $\hbar=1$. Using $\hbar$ =1.054571817 × 10^{-34} Js [50], changing the unit of λ to Joule, and dividing λ with the value of $\hbar$, we find the correspondence of these parameters.

From these results, we know that the value of both parameters are close to zero relative to $\zeta=1$ or $\lambda\to\infty$. These values are in accordance with other works which get the value of the decoherence parameter close to zero [17], [21], [29]. It means the interaction between the system and environment does not affect the system significantly. Therefore, quantum properties in the system related to the entanglement of the strangeness are preserved.

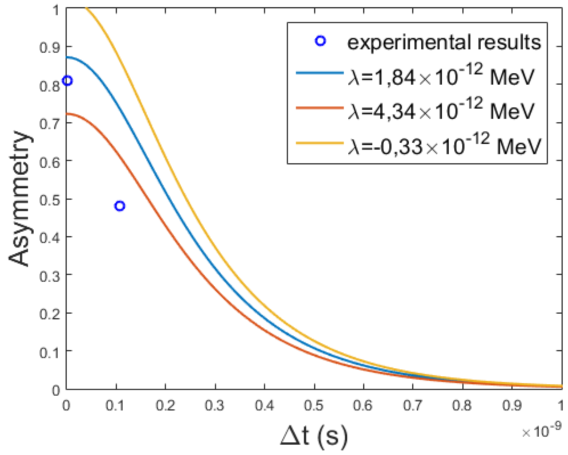

Fig. 1. Asymmetry is plotted versus time measurement difference based on experimental results and the value of λ

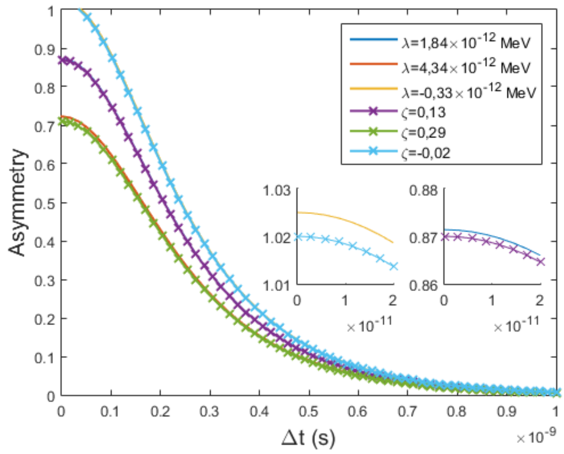

Fig. 2. Asymmetry is plotted versus time measurement difference based on the value of λ and ζ. Their value (including upper and lower limit) are corresponding to each other

CONCLUSION

In this paper, the dynamical equation of entangled K-meson has been determined based on the Lindblad equation. From this model, we know that, for $t > 0$, decoherence phenomenon occurs. This phenomenon's influence is represented by decoherence parameter (λ) and effective parameter decoherence (ζ). Using Standard Least-Squares method, we get $\zeta = 0.13 \pm 0.865$. This result is in accordance with the results of the references that use the effective variance method, $\zeta = 0.13^{+0.16}_{-0.15}$. We have shown that $\zeta = 0.13^{+0.16}_{-0.15}$ corresponds with $\lambda = (1.84^{+2.50}_{-2.17}) \times 10^{-12}$ MeV. The value of both parameters are close to zero relative to $\zeta = 1$ or $\lambda \to \infty$. It means that the interaction between system and environment does not affect the system significantly. Therefore, quantum properties in the system related to the entanglement of the strangeness are preserved.

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