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GW170817 Implementation on Einstein-Gauss-Bonnet Theory with Non Minimal and Non Minimal Derivative Coupling

F Khairunnisa[1] , G Hikmawan[1] and F P Zen[1,2]

[1]*Theoretical Physics Laboratory, Department of Physics, Institut Teknologi Bandung, Jl. Ganesha 10 Bandung, Indonesia.*

[2]*Indonesia Center for Theoretical and Mathematical Physics (ICTMP), Institut Teknologi Bandung, Jl. Ganesha 10 Bandung, 40132, Indonesia.*

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Abstract

The GW170817 event manifests that gravitational wave velocity is close to the speed of light. As a result, several theories of gravity are no longer applicable, including Einstein-Gauss-Bonnet (EGB) inflation. However, a constraint equation could be applied so that the theory could produce a viable result. In this study, the EGB inflation is being extended by adding a non-minimal coupling (NMC) and a non-minimal derivative coupling (NMDC). Free parameters values were evaluated to obtained viability with observational indices. We use power law and exponential Gauss-Bonnet coupling functions. Each model provides observational values of n_s and r that are compatible with the observations and has its characteristic. It specifies the free parameter that controls the alteration of n_s and r values. The power-law model is controlled by the power m of the Gauss-Bonnet coupling function and the potential integration constant, V_2. While the exponential model is controlled by the potential integration constant c and the power m of the exponential function. Some approximations do not hold true so that the models need to be rectified. Apparently, the rectified power-law model is violating null energy condition (NEC), so we also provide the non-violating NEC power-law model.

Keywords: GW170817, Einstein-Gauss-Bonnet, inflation, NEC.

INTRODUCTION

As the inflation theory proposes, the universe should encounter an era of accelerated expansion to solve several problems of the former Big Bang theory [1]. The most well-known theory is slow-roll inflation in which the acceleration of the universe is generated by a scalar field that is slowly rolling at its potential [2], this inflation theory is still progressing until recently. The rapid expansion of the universe at $t \simeq 10^{-35}$s which is associated with the shrinking of Hubble radius is affecting some cosmological scales of interest to freeze out and re-enter at some later times. Cosmological perturbation which appeared as CMB fluctuations was created during this era, about $N = 60$ e-folds before the end of inflation [3]. The essential tools related to inflation is the comoving Hubble radius, $(aH)^{-1}$, or Hubble horizon. Any scales of interest are able to contact each other inside this horizon (subhorizon) and ”freeze out” outside (superhorizon). Former Big Bang scenario followed this path until Cosmic Microwave Background observation shed path in history. This radiation was first detected in 1965 by Penzias and Wilson [5] as the relic of our infant universe. From the map, we realize that our observational universe is not as homogeneous and isotrophy as we once believed. The problem started to rise, how could some scales of interest that never contacted are sharing a nearly equal temperature. Aforementioned scenario failed toexplain this. There has to be an era where these scales ever contacted. Then inflation scenario came to the ”rescue” as the seed of the scenario. Briefly,

* Corresponding author.
E-mail address: fkphy1996@gmail.com

cosmological perturbation started their lives in sub-horizon scales [3], $k \gg (aH)^{-1}$, and freeze out once they on superhorizon scales, $k \ll (aH)^{-1}$ so that they ever contacted in the past, out of our ability to see. As mentioned before, these scales re-rentry at some later times carrying information from the earlier universe. Since inflation ”stretched” our universe to earlier time [3], means that our observational universe is no longer ”start” at t = 0 but even further before to give chance of cosmological scales to contact. However, this theory is still speculative that we could attempt various approaches to obtain viability. Another scenario of inflation are feasible such as quantum entanglement in inflation as reader is pleased to check in refs. [7, 8, 9].

The CMB fluctuations have been considered to have started during the inflationary era. The gravitational wave that resonates with tensor perturbation in the early universe is investigated in this work. We'll be presenting the recent notable event involving gravitational waves. The binary neutron star merger was discovered by the LIGO and Virgo inferometers in August 2017 [10]. The GW170817 event revealed that gravitational wave speed is close to the speed of light, implying that gravitons are nearly massless. This is due to the fact that this event occurred at almost simultaneously as a gamma-ray burst (GRB) GRB170817A, which was observed independently by Fermi detectors [11]. The observed time delay between the two was 1.74 ± 0.05 s [10]. Hence it ruled out several modified gravity theories that contradict the event (see ref. [11] for reviews) including Einstein-Gauss-Bonnet inflation [14, 24, 12, 13]. However, several works [12, 13, 14] indicated that the theory are able to obtain viability after a certain constraint imposed. Einstein-Gauss-Bonnet theory is an appealing candidate because it is string corrected theory [12, 13, 14].

In this work, we are tempted to modify this theory by including a non-minimal coupling (NMC) and a non-minimal derivative coupling (NMDC) to the action and see the behaviour that might arise. The prior was first proposed by Amendola in 1993 [15, 16]. According to [15], NMC term has been enganged to production of oscillating Universe, allows to solve the ”graceful exit” problem of the old inflation and many other employment as ca be seen in [15] and references therein. As for the NMDC, it was proposed by Granda in 2010 [17]. The interesting thing is NMDC term potrays as a dark matter at early stage [17, 16]. Another examples of using this term can be checked in [16].

In fact these type of theory belong to the general scalar-tensor Horndeski theory [18, 19, 20]. However, Horndeski theories also flawed to describe phenomonology of GW170817 since it produced gravitational wave speed less than the speed of light [13, 11]. We will soon reveal that the theory is still capable to describe the evolution of universe. We use effective field theory approach to study perturbation view. This theory is useful to deal with low-energy degrees of freedom present for inflation [18]. It is showed that general Horndeski theory belongs to the action of EFT framework [18, 20]. This theory also provide distinct form of power spectrum from standar single-field slow-roll inflation model. However, this theory allows the speed of scalar and tensor perturbation other than the speed of light. We then apply $c_T = c = 1$ in natural units, which agrees with the previously mentioned event.

In section 2, we reconstruct the action consisting six terms including Einstein-Gauss-Bonnet and is followed by non-minimal and non-minimal derivative coupling then derive the equation of motion. In section 3 we compute the perturbative terms in EFT framework, employing Horndeski's theory. We proceed to expand the action up to second order in Hordenski lagrangian so that we could get equation for gravitational wave speed from the tensor perturbation. In the subsequent section 4 we incorporate GW speed constraint into the dynamics that have been built and using this we obtain constraint equation in term $\dot{\phi}$ which is applied to background equations and slow-roll indices. This method has been used in refs. [14, 24, 12, 13]. Slow-roll indices arise from the dynamics of background and fluctuations dynamics. In section 5, given two models of Gauss-Bonnet scalar function and study their behaviour in agreeing with observational indices. We also present about the null-energy-condition violation that might arise in the models in section 8.

MODEL RECONSTRUCTION

Consider the following action

* Corresponding author.
E-mail address: fkphy1996@gmail.com

$$S=\int d^4x\sqrt{-g}\,\Big(\frac{R}{2}-\frac{1}{2}\,g^{\mu\nu}\partial_{\mu\phi}\partial_{\nu\phi}-V(\phi)+\frac{1}{8}\,f(\phi)R_{GB}^2-\frac{1}{2}\zeta\phi^2R+\xi\phi G_{\mu\nu}\phi^{;\mu\nu}\Big), \quad (1)$$

the first term of (1) is the Einstein-Hilbert term where R is a Ricci scalar and we have used $M_{pl}^2=1/(8\pi G)=1$ in natural unit. The next two terms are kinetic and potential part of scalar field, $f(\phi)$ is a Gauss-Bonnet coupling scalar function, while $R_{GB}^2=R_{\mu\nu\rho\sigma}R^{\mu\nu\rho\sigma}-4R_{\mu\nu}R^{\mu\nu}+R^2$ describes the Gauss-Bonnet invariant with $R_{\mu\nu\rho\sigma}$ and $R_{\mu\nu}$ are Riemann tensor and Ricci tensor, respectively. The last two terms are NMC coupling and NMDC coupling with ζ and ξ being its coupling constant, respectively. This type of model also studied in ref. [25]. In this work, we assume the background metris is flat FLRW (Friedmann-Lemaitre-Robertson-Walker)

$$ds^2\;=\;-dt^2\;+\;a^2(t)\delta_{ij}dx^i\,dx^j \quad (2)$$

with the metric component reads as $g_{\mu\nu}=diag\left(-1,a^2(t),a^{2(t)},a^2(t)\right)$.

We need to vary action (1) with respet to the metric and the scalar field to get the equation of motion. The resulting energy momentum tensor are

$$\begin{aligned}
&G_{00}\;=\;3H^2\,,\\
&Gij\;=\;a^2\delta ij\left(-2\dot{H}-3H^2\right),\\
&T(\phi)_{00}=\frac{1}{2}\,\dot{\phi}^2+V(\phi),\\
&T(\phi)_{ij}=a^2\,\delta ij\left(\frac{1}{2}\,\dot{\phi}^2\;-\;V(\phi)\right),\\
&T(f)_{00}=-3H^3\dot{f},\\
&T(f)_{ij}\;=\;-a^2\,\delta_{ij}\left[-H2\ddot{f}-2\left(\dot{H}\;+\;H^2\right)\dot{H}f\right], \quad (3)\\
&T(\zeta)_{00}\;=\;3H^2\,\zeta\phi^2\;+\;6\zeta H\phi\dot{\phi},\\
&T(\zeta)_{ij}\;=\;a^2\,\delta_{ij}\zeta\phi^2\left(-2\dot{H}\;-\;3H^2\right)-\zeta a^2\,\delta_{ij}\left(2\dot{\phi}^2\;+\;2\phi\ddot{\phi}+\;4H\phi\dot{\phi}\right),\\
&T(\xi)_{00}\;=\;-9\xi H^2\dot{\phi}^2\,,\\
&T(\xi)_{ij}\;=\;a^2\,\delta_{ij}\xi\left(3H^2\dot{\phi}^2\;+\;4H\dot{\phi}\ddot{\phi}+\;2\dot{H}\dot{\phi}2\;\right).(3)
\end{aligned}$$

Therefore varying gravitational action eq. (1) with respect to the spacetime, we obtained the equation of motions as follows

$$3H^2\left(1+H\dot{f}\right)=\frac{1}{2}\,\dot{\phi}^2+V+3H^2\zeta\varphi^2\;+6\zeta H\varphi\dot{\varphi}-9\xi H^2\dot{\phi}^2\,, \quad (4)$$

$$2H^{\cdot}\;=(2\zeta-1\,)\dot{\phi}^2+H\left(H^2\dot{f}-2\dot{H}f-H\ddot{f}\right)--2\zeta\phi\left(H\dot{\phi}-\dot{H}\phi-\ddot{\phi}\right)+2\xi\dot{\phi}\left[\left(3H^2-\dot{H}\right)\dot{\phi}-2H\ddot{\phi}\right]. \quad (5)$$

Then Euler-Lagrange equation of lagrangian eq. (1) with respect to the scalar field are calculated to yield

$$\begin{aligned}
&(1-6\xi H^2)\left(\ddot{\phi}+3H\dot{\phi}\right)+V'\\
&-3f'H^2\left(\dot{H}+H^2\;\right)+6\zeta\phi\left(2H^2+\dot{H}\;\right) \quad (6)\\
&-12\xi H\dot{H}\phi=0
\end{aligned}$$

The dot and prime represent differentiation with respect to cosmic time and scalar field, respectively. Furhtermore, eqs. (4)-(6) can be simplified hence become easier to be solved analytically, thus we shall impose slow-roll approximations which read

$$\dot{H}\ll H^2\,,\dot{\varphi}^2\ll V,\ddot{\varphi}\ll H\dot{\phi} \quad (7)$$

Once these conditions imposed, we obtained following equations

$$3H^2\left(1+2F-3E'\dot{\phi}^2+H\dot{f}\right)=\frac{1}{2}\,\dot{\phi}^2+V-6H\dot{F}\,,$$

$$2\dot{H}\left(1+2F+\dot{H}f+\dot{E}\dot{\phi}\right)=\;-\dot{\phi}^2-H^2\ddot{f}+H^3\dot{f}+2H\dot{F}\;-2\ddot{F}+6H^2\dot{E}\;\dot{\phi}-\;4H\ddot{E}\phi\,, \quad (8)$$

$$\ddot{\phi}+3H\dot{\phi}+V'-3f'H^4-3f'H^2\dot{H}\;-12F'H^2-72F'\dot{H}-6E'H^2\ddot{\phi}-12E'H\dot{H}\dot{\phi}-18E'H^3\dot{\phi}=0,$$

where we have employed following redefinition

$$F(\phi)=-\tfrac{1}{2}\,\zeta\varphi^2E(\phi)=\xi\phi \quad (9)$$

Hence $\dot{F}=-\zeta\phi\dot{\phi}$, $\ddot{F}=-\zeta\dot{\phi}^2-\zeta\phi\ddot{\phi}$ and $\dot{E}=\xi\phi,{}^{\cdot}E^{\cdot\cdot}\;=\;\xi\varphi^{\cdot\cdot}$.

However, the simplified eq. (8) are still remain unsolvable, more approximations are needed then we can examine the viability of the models [24, 12]. Based on refs. [13, 12], Gauss-Bonnet coupling function can be neglected to attain more manageable equations. Then since we have performed redefiniton to some functions, one can impose additional approximations analogous to slow-roll approximation,

$$\ddot{F}\ll\;H\dot{F}\,,\ddot{E}\ll\;H\dot{E} \quad (10)$$

With this, (7) and (10) in hand, we obtained

$$3H^2(1+2F) \simeq V - 6H\dot{F} \quad (11)$$

$$2\dot{H}(1+2F) \simeq -\dot{\phi}^2 + 2H\dot{F} + 6H^2\dot{E}\dot{\phi} \quad (12)$$

$$(3H - 18H^3E')\dot{\phi} + V' - 12H^2F' \simeq 0 \quad (13)$$

or

$$H^2\,(1-\zeta\phi^2) \simeq \frac{1}{3}\,V + \,2H\zeta\phi\dot{\phi} \quad (14)$$

$$\dot{H}(1-\zeta\phi^2) \simeq -\frac{1}{2}\,\dot{\phi}^2(1-6\xi H^2) - H\zeta\phi\dot{\phi} \quad (15)$$

$$V' + 12H^2\zeta\phi + (3H - 18H^3\xi)\dot{\phi} \simeq 0, \quad (16)$$

where we have written the functions back to the equations. We will soon check the validity of foregoing approximations later in section 5.

DYNAMICS OF SCALAR AND TENSOR PERTURBATION

It is vital to address perturbation theory when dealing with inflation since it manifests the things that relevant to observation. It is also important to stress that inflationary tensor fluctuations of the FRW metric background arise from inflationary tensor fluctuations of the field itself, via mechanism that is similar to which leading to their scalar counterparts.

To consider the perturbation, we work in ADM (Arnowitt-Deser-Misner) formalism. In the presence of scalar field ϕ, we can chose unitary gauge $\delta\phi = 0$. This constant scalar field lays on constant time hypersurface [18]. Generally, ADM metric has the form

$$ds^2 = -N^2dt^2 + h_{ij}\big(dx^i + N^i\,dt\big)\big(dx^j + N^jdt\big), \quad (17)$$

where N is the lapse function, Ni is shift vactor and hij is the three-dimensional metric. Flat FRW metric follows the form $ds^2 = -N^2(t)dt^2 + a^2\delta_{ij}dx^idx^j$.

General gravitational theories depend on scalar quantities appearing in the ADM formalism such that

$$S = \int d^4x\,\sqrt{-g}L(N,K,\mathcal{S},\mathcal{R},\mathcal{Z},\mathcal{U};t). \quad (18)$$

The field kinetic term [18], $X \equiv g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi$, depends on N and t. The field ϕ participates the equation of motion through the partial derivatives $L_N \equiv \partial L/\partial N$ and $L_{NN} \equiv \partial^2L/\partial N^2$ [18]. In this framework we consider Maldacena gauge to fix time and spatial reparametrizations [26]

$$h_{ij} = a^2(t)e^{2\Theta}\left(\delta_{ij} + \gamma_{ij} + \frac{1}{2}\,\gamma_{il}\gamma_{lj}\right), \quad (19)$$

where $a(t)$, Θ, γ are scale factor, scalar perturbation and tensor perturbation, respectively. The latter is a traceless and divergence-free tensor such that $\gamma_{ii} = \partial_i\gamma_{ij} = 0$. It has two modes of polarization,

$$\gamma_{ij} = h_+e^+_{ij} + h_\times e^\times_{ij}\,. \quad (20)$$

In Fourier space, the transverse and traceless tensors e^+_{ij} and $e^\times_{ij}$ satisfy normalization condition such that $e_{ij}(\boldsymbol{k})e_{ij}\,(-\boldsymbol{k})^* = 2$ for each polarization, while $e^+_{ij}(\boldsymbol{k})e^\times_{ij}\,(-\boldsymbol{k})^* = 0$ [18]. Scalar and tensor perturbation can be analyzed separately due to decomposition theorem. Expansion of Lagrangian in eq. (18) up to second order is

$$L = \bar{L} - \dot{F} - 3HF + \big(\dot{F} + L_N\big)\delta N + E\delta_1R + \left(\frac{1}{2}L_{NN} - \dot{F}\right)\delta N^2 + \frac{A}{2}\delta K^2 + B\delta K\delta N + C\delta K\delta_1R + D\delta N\delta_1R + E\delta_2R + \frac{G}{2}\delta_1R^2 + L_S\delta K_{\mu\nu}\,\delta K^\nu_\mu + L_Z\delta R^\mu_\nu\delta R^\nu_\mu\,, \quad (21)$$

where

$$A = L_{KK} + 4HL_{SK} + 4H^2L_{SS},$$

$$B = L_{KN} + 2HL_{SN},$$

$$C = L_{KR} + 2HL_{SR} + \frac{1}{2}L_U + HL_{KU} + sH^2L_{SU},$$

$$D = L_{NR} - \frac{1}{2}\,\dot{L}_{NU}, \quad (22)$$

$$G = L_{RR} + 2HL_{RU} + H^2L_{UU}.$$

Moreover it is useful to express the Lagrangian in the Lagrangian density form so that the scale factor makes its appearance. In ADM formalism Lagrangian density is expressed as $\mathcal{L} = \sqrt{-g}L = N\sqrt{h}L$, where h is the determinant of three dimensional metric h_{ij} . Finally we can express

Lagrangian density in second order in following manners

$$L_2 = \delta\sqrt{h}\,[(F + L_N)\delta N + E\delta_1 R] + a^3\,\Big[\Big(L_N + \tfrac{1}{2}L_{NN}\Big)\delta N^2 + E\delta_2 R + \tfrac{A}{2}\delta K^2 + B\delta K\delta N + C\delta K\delta_1 R + (D+E)\delta N\delta_1 R + \tfrac{G}{2}\delta_1 R + L_S\delta K^{\mu}_{\nu}\delta K^{\nu}_{\mu} + L_Z\delta R^{\mu}_{\nu}\delta R^{\nu}_{\mu}\Big] \quad (23)$$

where we have used $\delta(N\sqrt{h}L = \sqrt{h}L\delta N + NL\delta h + N\sqrt{h}\delta L$ and at the and of calculation we set $N = 1$.

In 1993, Horndeski [19] proposed a work in which constructing Lagrangian with higher order terms which leads to second order field equation. This work is equivalent with Generalized Ginflation which associates to Galileon symmetry breaking when the spacetime is curved [21, 20]. Furthermore, these theory belong to EFT framework. In this work, we can exploit Horndeski theory since it is the most general scalar-tensor theory as we've mentioned earlier. The action of Horndeski theory defined as [19, 18, 20]

$$S = \int d^4x\,\sqrt{-g}\,X\,\Sigma_{1=2}^{5}\,L_i\,, \quad (24)$$

where

$$L_2 = G_2(\phi, X),$$

$$L_3 = G_3(\phi, X)\phi, \quad (25)$$

$$L_4 = G_4(\phi, X)R - 2G_{4X}(\phi, X)[(\phi)^2 - \phi^{;\mu\nu}\phi_{;\mu\nu}]\,,$$

$$L_5 = G_5(\phi, X)G_{\mu\nu}\phi^{;\mu\nu} + \frac{1}{3}G_{5X}\,[(\phi)^3 - 3(\phi)\phi_{;\mu\nu}\phi^{\mu\nu} + 2\phi^{;\mu\nu}\varphi_{\mu\sigma}\varphi^{;\nu}_{;\sigma}\,]\,,$$

w h e r e □ i s t h e a f o r e m

$$G_2 = \frac{1}{4}f''''X^2\left[3 - \ln\left(-\frac{X}{2}\right)\right],$$

$$G_3 = -\frac{1}{4}f'''X\left[7 - 3\ln\left(-\frac{X}{2}\right)\right], \quad (26)$$

$$G_4 = -\frac{1}{4}f''X\left[2 - \ln\left(-\frac{X}{2}\right)\right],$$

$$G_5 = -\frac{1}{2}f'\ln\left(-\frac{X}{2}\right).$$

With these in hands, eq. (1) can be translated by Horndeski language with the functions

$$G_2 = -\frac{X}{2} - V + \frac{1}{4}f''''X^2\left[3 - \ln\left(-\frac{X}{2}\right)\right],$$
$$G_3 = \frac{1}{4}f'''X\left[7 - 3\ln\left(-\frac{X}{2}\right)\right],$$
$$G_4 = \frac{1}{2} - \frac{1}{2}\zeta\phi^2 - \frac{1}{4}f''X\left[2 - \ln\left(-\frac{X}{2}\right)\right],$$
$$G_5 = \xi\phi - \frac{1}{2}f'\ln\left(-\frac{X}{2}\right). \quad (27)$$

Subsequently, we will see the emergence of scalar and tensor fluctuations. These fluctuations can be treated separately due to decomposition theorem [4]. The vector part is ignored since it is diluted away when acceleration expansion happened [4, 21].

3.1 Scalar Perturbation

On using eq. (19) while ignoring tensor part, dynamical of scalar perturbation are obtained by calculating eq. (23) and its components,

$$L_s = a^3 Q_s\left[\dot{\Theta}^2 - \frac{c_s^2}{a^2}\right] \quad (28)$$

$$Q_s = \frac{2L_S}{3W^2}(9W^2 + 8L_s w) \quad (29)$$

$$c_s^2 = \frac{2(\dot{M} + HM - E)}{Q_s} \quad (30)$$

where Q_s and c_s^2 are the auxiliary function. Nevertheless, notice that lagrangian (28) is the wave equation, thus c_s is considered as speed of sound of scalar perturbation.

The quantities in eqs. (28)-(30) are

$$L_{\mathcal{S}} = \frac{1}{2}(1 - \zeta\phi^2 + Hf'\dot{\phi} + \xi\dot{\phi}^2),$$

$$w \simeq 3\left[-3H^2(1-\zeta\phi^2) + 9H(\zeta\phi - Hf')\dot{\phi} + \left(\frac{1}{2} - 18H^3\xi\right)\dot{\phi}^2\right] \tag{31}$$

$$\mathcal{W} \simeq 2H\left(1 - \zeta\phi^2 + \frac{3}{2}Hf'\dot{\phi}\right) - 2\zeta\phi\dot{\phi} + 6H\xi\dot{\phi}^2,$$

$$\mathcal{M} = \frac{\left(1-\zeta\phi^2+\xi\dot{\phi}^2+Hf'\dot{\phi}\right)^2}{2H-2H\zeta\phi^2+3H^2f'\dot{\phi}-2\zeta\phi\dot{\phi}+6H\xi\dot{\phi}^2},$$

$$\mathcal{E} = \frac{1}{2}(1 - \zeta\phi^2 - \xi\dot{\phi}^2 + f''\dot{\phi}^2 + f''\ddot{\phi}),$$

where we have only considered up to second order and ignore higher orders.

3.2 Tensor Perturbation

On using perturbation in eq. (19) while ignoring scalar part, dynamical of tensor perturbation are obtained by calculating eq. (23) and its components,

$$\mathcal{L}_T = a^3 Q_T\left[\dot{\gamma}_{ij}^2 - \frac{c_T}{a^2}\left(\partial_{\gamma_{ij}}\right)^2\right]$$

$$Q_T \equiv \frac{L_{\mathcal{S}}}{2}, \quad c_T^2 \equiv \frac{\mathcal{E}}{L_{\mathcal{S}}} \tag{32}$$

Since traceless and divergence-free tensor has two-mode polarization as in eq. (20) we can proceed to express the lagrangian density in this mode term. Hence,

$$\mathcal{L}_T = a^3 Q_T\left[\dot{h}_\lambda^2 - \frac{c_T^2}{a^2}(\partial h_\lambda)^2\right], \tag{33}$$

where $\lambda = +, \times$ is each polarization mode.

3.3 Canonical Quantization

Furthermore we need to obtain scalar spectral index so that we are able to confront theory with observation. First, $\Theta(\tau, \boldsymbol{x})$ is expressed in Fourier mode and transformed such that

$$\Theta(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\,\hat{\Theta}(t,\vec{k})e^{i\vec{k}\cdot\vec{x}} \tag{34}$$

$$\hat{\Theta}(t,\vec{k}) = u(t,\vec{k})\hat{a}(\vec{k}) + u^*(t,-\vec{k})\hat{a}^\dagger(-\vec{k})$$

where k is the comoving wavenumber, $\hat{a}(\boldsymbol{k})$ and $\hat{a}^\dagger(\boldsymbol{k})$ are the annihilation and creation operators, respectively which obey commutation relation. Similarly, for tensor perturbation,

$$\gamma_{ij}(t,\vec{x}) = \int \frac{d^3k}{(2\pi)^3}\hat{\gamma}_{ij}(t,\vec{k})e^{i\vec{k}\cdot\vec{x}} \tag{35}$$

$$\hat{\gamma}_{ij} = \Sigma_{\lambda=+,\times}\,\hat{h}_\lambda e_{ij}^\lambda \tag{36}$$

$$\hat{h}_\lambda(t,\vec{k}) = h_\lambda(t,\vec{k})\hat{a}(\vec{k}) + h_\lambda^*(t,-\vec{k})\hat{a}^\dagger(-\vec{k}). \tag{37}$$

Then we proceed to express the Lagrangian (28) and (32) in these quantities. Finally the obtained equation of motion for each perturbation as follows, respectively

$$\ddot{u} + \left(3H + \frac{\dot{Q}_S}{Q_S}\right)\dot{u} + c_S^2\frac{k^2}{a^2}u = 0, \tag{38}$$

$$\ddot{h}_\lambda + \left(3H + \frac{\dot{Q}_T}{Q_T}\right)\dot{h}_\lambda + c_T^2\frac{k^2}{a^2}h_\lambda = 0, \tag{39}$$

These equations need to be expressed in conformal time. The rescaling such that $v = zu$ and $z = a\sqrt{2Qs}$ which has been studied in [18]. We proceed similar manners to both scalar and tensor. After the rescaling, eq. (28) becomes

$$v'' + \left(c_S^2k^2 - \frac{z''}{z}\right)v = 0, \tag{40}$$

which acknowledged as Mukhanov-Sasaki equation [3]. Notice that this equation is no longer has damping term like in the physical time form.

As universe appears to be quasi-de Sitter during inflation, we can employ de Sitter approximation where $\tau = -1/aH$ hence

$$\frac{z''}{z} \simeq \frac{1}{a}\frac{d}{d\tau}\,a^2H = -H\tau\frac{d\left(\frac{1}{H^2\tau^2}\right)}{d\tau} = \frac{2}{\tau^2} \tag{41}$$

Then eq. (40) is presented as below

$$v'' + \left(c_S^2k^2 - \frac{2}{\tau^2}\right)v = 0. \tag{42}$$

To obtain the solution of (42), we need to consider boundary condition as normalization solution. In early time where universe is considered vacuum, we can choose Bunch-Davies vacuum as the normalization solution [18],

$$v = \frac{e^{-ic_{sk}}}{\sqrt{2csk}}. \tag{43}$$

Then we got the complete solution as

$$v(\tau) = \frac{e^{-ic_s k\tau}}{\sqrt{2csk}}\left(1 - \frac{i}{c_s k\tau}\right) \quad (44)$$

Remember that $v = zu$, $z = a\sqrt{2Q_s}$, then eq. (44) becomes

$$u = -\frac{ie^{-ic_s k\tau}}{2a\tau(c_s k)^{\frac{3}{2}}\sqrt{Q_s}}(1 + ic_s k\tau) \quad (45)$$

de Sitter solution for mode u yields to

$$u = \frac{iHe^{-ic_s k\tau}}{2(c_{sk})^{\frac{3}{2}}\sqrt{Qs}}(1 + ic_s k\tau) \quad (46)$$

for each polarization mode. Following the same manner as scalar perturbation, solution for tensor mode yield to

$$h_\lambda = \frac{iHe^{-ic_T k\tau}}{2(c_T k)^{\frac{3}{2}}\sqrt{Q_T}}\,(1 + ic_T k\tau) \quad (47)$$

Since the solution for each perturbation has been defined, we then proceed to employ it to twopoint-correlation function which associated with power spectrum. Calculating the expectation value of Θ at vacuum and taking the solution of eq. (46) in the late-time and at superhorizon limit, then the scalar power spetrum is

$$P_\Theta = \frac{H^2}{8\pi^2 Q_s c_s^3}, \quad (48)$$

which is already scale-invariant. Then, the tilt of eq. (48) in the horizon crossing, $c_s k = aH$, where the cosmological scales of interest remain ”frozen” which read as

$$n_s - 1 \equiv \left.\frac{d\ln P_\Theta}{d\ln k}\right|_{c_s k = aH} = -2\frac{\dot{H}}{H^2} - \frac{\dot{Q}_s}{HQ_s} - \frac{3\dot{c}_s}{Hc_s}. \quad (49)$$

As for the tensor perturbation, we calculate the expectation value of eq. (35) to yield

$$\mathcal{P}_h = \frac{H^2}{2\pi^2 Q_T c_T^3} \quad (50)$$

The tilt of the tensor power spectrum is a bit different than that was defined in scalar. The scale invariant tensor spectrum is defined when nT = 0 instead of equal to one. The tilt of eq. (50) or tensor spectral index at horizon crossing then

$$n_T \equiv \left.\frac{d\ln P_h}{d\ln\ k}\right|_{c_T k = aH} = -\frac{2\dot{H}}{H^2} - \frac{\dot{Q_T}}{HQ_T} - 3\frac{\dot{c_T}}{Hc_T} \quad (51)$$

CONSTRAINT OF GW SPEED

In Horndeski function, eq. (27), the tensor propagation speed in eq. (32) can be written as

$$c_T^2 = \frac{\left(G_4 + \frac{1}{2}XG_{5\phi} - XG_{5X}\ddot{\phi}\right)}{G_4 - 2XG_{4X} - H\dot{\phi}XG_{5X} - \frac{1}{2}XG_{5\phi}} \quad (52)$$

where the subscript stands for the derivation with respect to quantity of interest, for example $G_{5X} = \frac{dG_5}{dX}$ which also applied to the scalar field. Using eq. (27), eq. (52) becomes

$$c_T^2 = \frac{1 - \zeta\phi^2 - \xi\dot{\phi}^2 + f''\dot{\phi}^2 + f''\ddot{\phi}}{1 - \zeta\phi^2 + \xi\dot{\phi}^2 + Hf'\dot{\phi}}, \quad (53)$$

where we have used $X = -\dot{\phi}^2$. Equating eq. (53) to unity to fullfil observational contraint we have mentione earlier, $c_T^2 \simeq c = 1$ in natural unit, we got the constraint equation,

$$f''\dot{\phi}^2 + f'\ddot{\phi} - Hf'\dot{\phi} - 2\xi\dot{\phi}^2 = 0 \quad (54)$$

then we can apply slow-roll condition in eq. (7) to eq. (54). Slow-roll approach implies $f'\ddot{\phi} \ll f'H\dot{\phi}$ then we get the constraint equation as follows

$$\dot{\phi} = \frac{Hf'}{f'' - 2\xi}. \quad (55)$$

This method was previously used by [14, 24, 12, 13]. We apply (55) to eq. (14) to yield

$$H^2 \simeq \frac{1}{3}V\,\left[\frac{f'' - 2\xi}{(1 - \zeta\phi^2)(f'' - 2\xi) - 2\zeta f'\phi}\right], \quad (56)$$

$$\dot{H} \simeq -\frac{H^2}{2(1 - \zeta\phi^2)}\left(\frac{f'}{f'' - 2\xi}\right)^2\left[1 + \zeta\phi\left(\frac{f'' - 2\xi}{f'}\right)\right] \quad (57)$$

$$V' + \left[\frac{4\zeta\phi(f'' - 2\xi) + f'}{(1 - \zeta\phi^2)(f'' - 2\xi) - 2\zeta f'\phi}\right]V \simeq 0, \quad (58)$$

Referring to refs. [13, 12], approximations $f'H^4 \ll V'$ can be applied so that we can attain a more managable potential differential equation [12]. Hence we can use this analogy to neglect $E'H^3\dot{\phi}$ from equation (13) which also contain H^4 term due to the form of $\dot{\phi}$ from eq. (55) so that eq. (56)-(58) are obtained.

To confront the theory with the observation, we need some quantities that relates theoretical indices to observational indices. The latter is described by scalar spectral index, ns, and the tensor-to-scalar ratio, r, while tensor spectral index, n_T , have yet to be observed [12]. From eq. (49) and (51) we can define the dynamics of inflation that ruled by these slow-roll indices:

$$\epsilon = -\frac{\dot{H}}{H^2}, \qquad \eta = \frac{\ddot{\phi}}{H\dot{\phi}},$$

$$\delta Q_s \equiv \frac{\dot{Q}_s}{HQ_s}\ , \ \ \delta Q_T \equiv \frac{\dot{Q}_T}{HQ_T}, \tag{59}$$

$$\delta c_s \equiv \frac{1}{2Hc_s^2}\frac{d}{dt}\ c_s^2\ , \ \ \delta c_T \equiv \frac{1}{2Hc_T^2}\frac{d}{dt}\ c_T^2\ ,$$

where the first and the second of eq. (59) was naturally arised by slow-roll inflation nature, while the rests of it were arised from second-order dynamics that we derived in sectopn. 3. It is reasonable to define additional slow-roll indices in such manners. Note that, according to observation the observed indices are [27]

$$n_s = 0.9649 \pm 0.0042,\ r\ <\ 0.056, \tag{60}$$

where r is tensor-scalar ratio. Notice that in eq. (60), scalar spectral index is nearly scale invariant. Hence, the RHS of eq. (49) have to be really small compared. Meanwhile, tensor spectral index have yet to be observed [36, 24]. Hence we approach it to also be nearly scale invariant.

When we plug eq. (55) into eq. (16), we get H2 term then we substitute eq. (56) into it so that eq. (58) is obtained. This shall be done since we want to form a differential equation for V to be used further. As can be seen in eq. (58), each scalar couplings, scalar potential and scalar field are interconnected in one differential equation. This also studied in refs. [14, 24, 12, 13]. The unknown function here is $f(\phi)$ which belongs to Gauss-Bonnet term, while others are constant couplings we've mentioned earlier. Therefore given a function $f(\phi)$, then we are able to solve differential eq. (58). We've been discussing about eqs. (56) and (58), now we proceed to eq. (57). It naturally arises the first slow-roll index we wrote in (59)

Now we proceed to apply eq. (55) to the quantities in eq. (59), we conclude that

$$\epsilon = \frac{1}{2(1-\zeta\phi^2)}\left(\frac{f'}{f''-2\xi}\right)^2\left[1-2\zeta\phi\left(\frac{f''-2\xi}{f'}\right)\right]$$

$$\eta = \frac{f''}{f''-2\xi} - \epsilon - \frac{(f'''f')}{(f''-2\xi)^2},$$

$$\delta Q_T = \frac{(-2\zeta\phi + 2\eta\xi H + (\eta-\epsilon)H^2 f')f'(f''-2\xi) + H^2 f'' f'^2}{(f''-2\xi)[(1-\zeta\phi^2)(f''-2\xi) + \xi H f' + H^2 f'^2]} \tag{61}$$

While the rest of indices in eq. (59) are not written explicitly. But we will soon show some quantities that still in reach analytically (note that in calculation we've only considering ϕ˙ up until second order and ignoring higher orders). To obtain δ_{c_s} we need

$$Q_s = 6L_{\mathcal{S}} + \frac{4}{3}\frac{(\mathcal{M}w)}{\mathcal{W}}$$

$$\dot{Q}_s = 6L_{\mathcal{S}} + \frac{4}{3}\left[\frac{(\dot{w}\mathcal{M} + w\dot{\mathcal{M}})\mathcal{W} - w\mathcal{M}\dot{\mathcal{W}}}{\mathcal{W}^2}\right] \tag{62}$$

Their components are calculated as follows

$$\dot{L_{\mathcal{S}}} = \frac{1}{2}\left(0 - 2\zeta\phi\dot{\phi} + 2\xi\dot{\phi}\ddot{\phi} + \dot{H}\dot{f} + H\ddot{f}\right),$$

$$\begin{aligned}\dot{w} = 3\{&-6H\dot{H} + \dot{\phi}\ddot{\phi} \\ &+ 6\left[\zeta\left(H\dot{H}\phi^2 + H^2\phi\dot{\phi} + H\dot{\phi}^2 + \dot{H}\phi\dot{\phi} + H\phi\ddot{\phi}\right)\right. \\ &- \left(3H^2\dot{H}\dot{f} + H^3\ddot{f}\right)6\xi\left(H\dot{H}\phi^2 + 2H^2\dot{\phi}\ddot{\phi}\right)\}\end{aligned}$$

$$\begin{aligned}\dot{\mathcal{W}} = 2\dot{H} - 2\zeta(&H\dot{\phi}^2 + 2H\phi\dot{\phi} + \dot{\phi}^2 + \phi\ddot{\phi}) \\ &+ 6\xi(\dot{H}\dot{\phi}^2 + 2H\dot{\phi}\ddot{\phi}) + 3(2H\dot{H}\dot{f} + H^2\ddot{f})\end{aligned} \tag{63}$$

$$\dot{\mathcal{M}} = 4\left(\frac{2L_{\mathcal{S}}L_{\dot{\mathcal{S}}}}{\mathcal{W}} - \frac{L_{\dot{\mathcal{S}}}^2}{\mathcal{W}^2}\right).$$

One can see the details about these quantities in refs. [18, 20, 22]. To obtain δ_{c_s} we need

$$c_s^2 \equiv \frac{2}{Q_s}\left(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}\right)\ \text{and}$$

$$\frac{d}{dt}c_s^2 = 2\left[\frac{\left(\ddot{\mathcal{M}} + H\mathcal{M} + H\dot{\mathcal{M}} - \dot{\mathcal{E}}\right)Q_s - \left(\dot{\mathcal{M}} + H\mathcal{M} - \mathcal{E}\right)\dot{Q}_s}{Q_s^2}\right] \tag{64}$$

Their components are

$$\ddot{\mathcal{M}} = \frac{4}{\mathcal{W}^4}\left[\left(2\dot{L_S}^2\mathcal{W} + 2L_S\ddot{L_S}\mathcal{W} - L_S^2\ddot{\mathcal{W}}\right)\mathcal{W}^2 - 2\mathcal{W}\dot{\mathcal{W}}\left(2L_S\dot{L_S}\mathcal{W} - L_S^2\mathcal{W}\right)\right],$$

$$\ddot{L_S} = \frac{1}{2}[-2\zeta(\dot{\phi}^2 + \phi\ddot{\phi}) + 2\xi\left(\ddot{\phi}^2 + \dot{\phi}\dddot{\phi} + 2\dot{H}\dot{f} + \ddot{H}\dot{f} + H\ddot{f}\right] \tag{65}$$

$$\ddot{\mathcal{W}} = 2\ddot{H} - 2\zeta\left[\ddot{H}\phi^2 + 2\left(\dot{H}\phi\dot{\phi} + \dot{H}\phi\dot{\phi}\right) + 3\dot{\phi}\ddot{\phi} + \phi\dddot{\phi}\right] + 6\xi\left[\ddot{H}\dot{\phi}^2 + 2\left(\dot{H}\dot{\phi}\ddot{\phi} + H\ddot{\phi}^2 + H\dot{\phi}\dddot{\phi}\right)\right] + 3[2(\dot{H}^2\dot{f} + H\ddot{H}\dot{f} + H\dot{H}\ddot{f}H\dot{H}\ddot{f} + H^2\dddot{f})$$

$$\dot{\mathcal{E}} = \frac{1}{2}\left[-2\zeta\left(\dot{\phi}^2 + \phi\ddot{\phi}\right) - 2\xi\left(\dot{\phi}\dddot{\phi} + \ddot{\phi}^2\right) + \dddot{f}\right],$$

To obtain δ_{c_T} , aside from eq. (53) we need

$$\frac{d}{dt}c_T^2 = \frac{d}{dt}\frac{1 - \zeta\phi^2 - \xi\dot{\phi}^2 + f''\dot{\phi}^2 + f''\ddot{\phi}}{1 - \zeta\phi^2 + \xi\dot{\phi}^2 + Hf'\dot{\phi}} . \tag{66}$$

Here we consider two auxiliary functions to make the calculation easier. Consider the numerator of eq. (66) is a and its denominator is b, so we can proceed

$$\frac{d}{dt}c_T^2 = \frac{\dot{a}b - a\dot{b}}{b^2} . \tag{67}$$

Their components were obtained as follows

$$\dot{a} = -2\left(\zeta + \eta H\xi\dot{\phi}\right)\dot{\phi} + \eta(\eta - \epsilon)H^2 f'\dot{\phi} + 3f''\eta H\dot{\phi}^2 , \tag{68}$$

$$\dot{b} = [-\,2\zeta\phi + (\eta - \epsilon)H^2 f']\,\dot{\phi} + (2\xi\eta + f'')H\dot{\phi}^2 .$$

Up until now, we have derived some functions required to the models ahead. In the next section, we also plot some variables and see the behaviours

MODELLING THE SCALAR FUNCTION

In this section we will derive some spesific models of a given scalar coupling function $f(\phi)$. Then we get the potential from eq. (58). We need an input of scalar field ϕ which we can get from numer of e-folds. By definition, e-folds is

$$N = \int_{t_i}^{t_f} H dt \tag{69}$$

which can be expressed in ϕ as

$$N = \int_{\phi_i}^{\phi_f} \frac{H}{\dot{\phi}} d\phi. \tag{70}$$

Substituting eq. (55) we obtained

$$N = \int_{\phi_i}^{\phi_f} \frac{f'' - 2\xi}{f'}\, d\phi. \tag{71}$$

We can obtained ϕ_f by equating in eq. (61) to unity, substitute it back to eq. (71) along with $N = 60$ then we can get ϕi as the input of scalar field ϕ [14, 12, 13].

5.1. Power Law Scalar Coupling of $f(\phi)$

Here we consider f(ϕ) and its derivatives has these form

$$f = \lambda\phi^m,\ f' = \lambda m\phi^{m-1} , \tag{72}$$

where λ is dimensionful constant and m is the power, then we can write its second derivative as

$$f'' = f'(m - 1)\phi^{-1} . \tag{73}$$

This model also studied in ref. [14]. To find the expression of potential, we use equation (58) but it turned out uneasy to solve. Here we consider $f'' \gg 2\xi$ to avoid non-linearity and its validity will be investigated later on. Then, make use of eq. (73), we yield following differential equation

$$V' + \left[\frac{4\zeta\phi f'(m-1)\phi - 1 + f'}{(1-\zeta\phi)f'(m-1)\phi - 1 + 2f'\zeta\phi}\right]V = 0, \tag{74}$$

and the resulting scalar potential is

$$V = V_2 \exp\left(\frac{(4(m-1)\zeta + 1)}{(2m-6)\zeta}\ln(|(m - 3)\zeta\phi^2 - m + 1|) + c\right), \tag{75}$$

where V_2 and c are integration constant. We also make use the approaches to the rest of eq. (14) to form

$$H^2 \simeq \frac{1}{3}V\left[\frac{(m-1)\phi^{-1}}{(1-\zeta\phi^2)(m-1)\phi - 1 + 2\zeta\phi}\right]$$

$$\dot{H} \simeq -\frac{H^2}{2(1 - \zeta\phi^2)}\left[\left(\phi(m - 1)\right)^2\left(1 - 2\zeta(m - 1)\right)\right]. \tag{76}$$

Then the slow-roll indices in eq. (61) become

$$\epsilon = \frac{1-2\zeta(m-1)}{2(1-\zeta\phi^2)}\left(\frac{\phi}{m-1}\right)^2$$

$$\eta = 1-\epsilon-\frac{m-2}{m-1}, \quad (77)$$

$$\delta_{Q_T} = \frac{(-2\zeta\phi+2\eta\xi H+(\eta-\epsilon)H^2 f')+H^2(m-1)f'}{[(1-\zeta\phi^2)(m-1)+\xi H+H^2 f']}.$$

where $f''' = (m-1)f'\phi^{-2}(m-2)$. The rest of slow-roll indices in (61) was not written analytically. We then proceed equating $\epsilon = 1$ to obtain final scalar field ϕ_f as

$$2(m-1)^2 - 2\zeta\phi_f^2(m-1)^2 = \phi_f^2\big(1-2\zeta(m-1)\big) \quad (78)$$

thas has solution

$$\phi_f = \pm\sqrt{\frac{2(m-1)^2}{1+2\zeta(m-1)(m-2)}}. \quad (79)$$

Then for ϕ_i as an input, we make use of previous approach for N to yield

$$N = (m-1)\int_{\phi_i}^{\phi_f}\frac{1}{\phi}\,d\phi \quad (80)$$

From this we get

$$\phi_i = \phi_f e^{-\frac{N}{m-1}} \quad (81)$$

to be the input for ϕ. Then with following example values $(\lambda, N, V2, \xi, \zeta, m, c) = (0.1,\ 60,\ 1328.28,$ -10^{-5} , -10^{-1} , 39.664, 0) then yields $n_s = 0.96465$ and $r = 0.0241$ which are accepted to recent observation values [27]. For the unobserved indices we obtained $n_T = -0.01948$ which is nearly scale invariant. Furthermore the initial and final scalar field values are $\phi_i = 0.6800$ and $\phi_f = 3.2095$, respectively. Finally we obtained slow-roll indices which have relatively very small values, $(\epsilon, \eta, \delta_{Q_S}, \delta_{c_S}, \delta_{Q_T}, \delta_{c_T})$ = (0.0013, 0.0246, 0.0666, −0.0113, 0.0162, 0.000229). We plot variations of m dan V2 for each value of ns and r in figure 1, we got the accepted values by searching throughout the plot using contour plot in MATLAB. Then variation of ξ and m while preserving other parameters are presented in figure 2.

We also conduct variation of one variable to see how it impact the output of observational indices. In this model, the one that give control to a good values of observational indices is m and V_2. For instance as present in table 1. Otherwise if we alter ξ and ζ it does give sigificant change, for example if ξ = (-10.10^{-5}, -5.10^{-5} , -2.10^{-5} , -13.10^{-3} , -10.10^{-3} , -10^{-3}) then the observational indices are (n_s, r) = (1.3276, 0.0136; 1.1755, 0.012; 1.0139, 0.017; 0.962, -14.85; 0.9704, -13.1305; 1.0483, 1.0659) which give poor values of them. Likewise, if ζ = (−1, −0.5, −0.2, 0.1, 0.5) then the observational indices are (n_s, r) = (0.9829, 0.057; 0.9829, 0.042; 0.9830, 0.0302; 1.2499, -0.013; 0.9946, -0.04) which doesn't give significant change around −0.1 and give poor value in positif sign of ζ. What we need to stress here and also for the subsequent models is, these controlling parameters are model-based and only give us the local solutions. Their alteration behaves monotonically in the accepted regions as we can see in the graph plots. This is due to the manual search and fitting method we have mentioned earlier, used in this paper. In the future, we will search another sophisticated method to obtain more precise values and visualization.

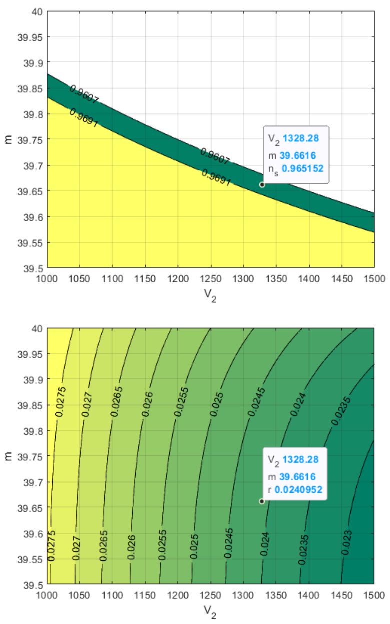


Fig. 1. The accepted value of n_s (left) in green shaded region based on Planck collab. 2018 [27] and accepted values of r (right) with respect to alteration of free parameters V_2 and m while preserving other parameters.

5.2 Exponential Scalar Function

For this model, scalar function has the form as below

$$f = e^{m\phi}\,, \tag{82}$$

where m is the power value. Its first and second derivatives are

$$f' = me^{m\phi},\ f'' = m^2 e^{m\phi} = mf'\,. \tag{83}$$

In this model we also apply where $f'' \gg 2\xi$. Then the potential scalar is

$$V = V_3 \exp\left[2\ln(|m(\zeta\phi^2 - 1) - 2\zeta\phi) + \frac{(4\zeta+1)\arctan\sqrt{\zeta}(m\phi-1)\alpha^{-1}}{\sqrt{\zeta}\alpha} + c\right] \tag{84}$$

where V_3 and c is are integration constants, and $\alpha = \sqrt{-\zeta - m^2}$. We have been using two integration constant to both model. This is reasonable as we will set either one is zero or neither. Applying to (9) we yield

$$H^2 \simeq \frac{1}{3}V\left[\frac{m}{(1-\zeta\phi^2)m + 2\zeta\phi}\right] \tag{85}$$

$$\dot{H} \simeq -\frac{H^2}{2m^2(1-\zeta\phi^2)}(1-2m\zeta\phi).$$

Therefore, some of slow roll indices in (61) become

$$\epsilon = \frac{1-2m\zeta\phi}{2m^2(1-\zeta\phi^2)}$$

$$\eta = -\epsilon \tag{86}$$

$$\delta_{Q_T} = \frac{-2\zeta\phi + 2\eta\xi H + H^2 m(me^{2m\phi} - 2\epsilon e^{m\phi})}{m(1-\zeta\phi^2) + H(\xi + Hme^{m\phi})}$$

By setting $\epsilon = 1$ we obtain the value of final scalar field,

$$\phi_f = \frac{1}{m^2}\left(\frac{1+2m}{4} - \frac{1-2m^2}{2\zeta}\right). \tag{87}$$

Using the same manner as previous model, initial scalar field value as follows

$$\phi_i = \frac{m\phi_f - N}{m} \tag{88}$$

For following free parameters value $(V_3, N, \zeta, \xi, m, c)$ = (104 , 60, 1, 10^{-10}, 59.2273, -11.4242), the obtained observational indices are $n_s = 0.96412$ dan $r = 0.02071$ which are accepted to latest Planck data [27], and tensor spectral index is $n_T = -0.02368$ which is nearly scale invariant. Initial and final scalar field values are $\phi_i = -0.004675$ and $\phi_f = 1.00837$, respectively. In this model we also obtained relatively small slow-roll indices, such as $(\epsilon, \eta, \delta_{Q_S}, \delta_{c_S}, \delta_{Q_T}, \delta_{c_T})$ = (0.0002215, -0.0002215, 1.4037, -0.4561, 0.0232, -0.0000025). Plot of varied values of m and V_2, respectively for n_s dan r is presented in figure 4.

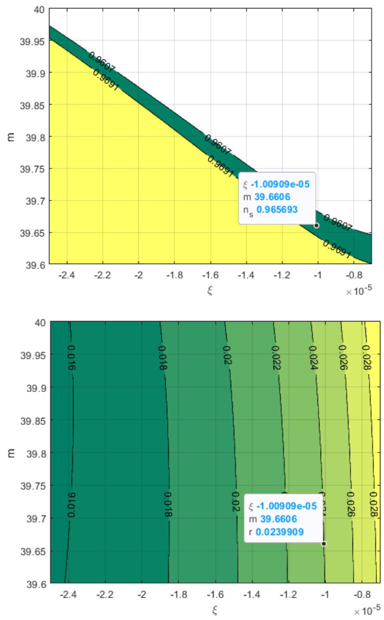
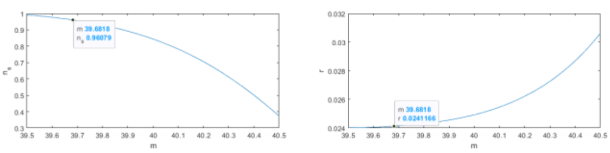


Fig. 2. The accepted value of n_s (left) in green shaded region based on Planck collab. 2018 [27] and accepted values of r (right) with respect to alteration of free parameters ξ and ζ while preserving other parameters.

Table 1. Values of n_s and r with respect to varying values of m and V_2.

m	n_s	r
10	0.994003	0.000011
15	0.98431	0.0006088
25	0.983799	0.00887
39.5	0.96075	0.02412
40	0.844069	0.02492

m	n_s	r
1000	0.9913	0.028
1200	0.9766	0.025
1300	0.9674	0.024
1400	0.9571	0.023
1500	0.9456	0.0228



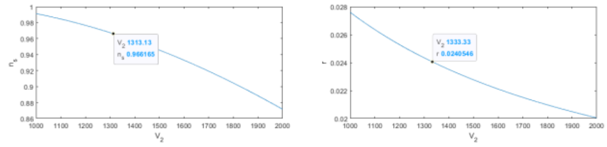


Fig. 3. Plot of n_s and r over varying (a) m within range [39.5, 40.5] and (b) V_2 within range [1000, 2000]. We see here that these parameters controlling the values of ns and r conveniently. Increasing value of V_2 give a decreasing value of both n_s and r at increasing rate and decreasing rate, respectively. On the other hand increasing value of m acts differently on n_s and r, respectively. As can be seen the prior gives decreasing value of n_s at an increasing rate while the latter gives increasing value of r at an increasing rate.

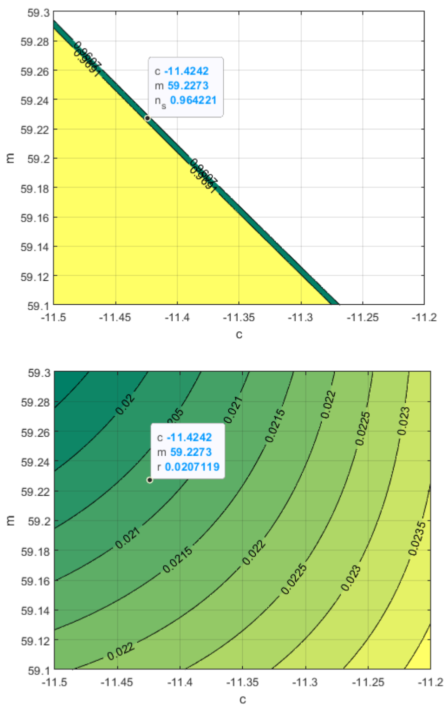


Fig. 4. The accepted value of ns (left) in green shaded region based on Planck collab. 2018 [27] and accepted values of r (right) with respect to alteration of free parameters m and c while preserving other parameters.

After some examination, we realize that the small alteration of ξ does not affect the change of observational indices, this might happened since we applied approachment $f'' \gg \xi$. Then we plot varying values of V_3 and ζ while preserving other parameters, as can be seen in table 2 and figure 5.

Plot of n_s with respect to V_3 and ζ are presented in figure 6. If other parameters are altered then we get random values of observational indices. For instance if $m = (40, 42, 43, 58, 60)$ then scalar spectral index and ratio tensor-to-scalar are (n_s, r) = (0.93876, 2.8632; 0.96158, 2.3016; 0.9716, 2.0576; 1.4369, 0.0583; -0.5525, 0.0293), we've seen here the slight change of m around 40 can alter the value of ns rigorously but at the same time can not provide proper value of r. Furthermore if $c = (-5, -12, 1)$ then (n_s, r) = (-0.0230, 4.88097; 1.52720, 0.01825; -0.000056, 5.3356), which are not expected. However, altering ξ tends to not giving any significant change of observational indices as long as its value is much smaller than one, $\xi \ll 1$, when its value is larger than one then we do not obtain corresponding value of observational indices. For an additional comment, the resulted aforementioned example values indicates that integration constant c has a value rather than zero. Thus we can express integration constant of V in equation (84) as

$$V_3' = V_3 e^c \,, \qquad (89)$$

where V_3 is the previous integration constant found in (84).

Table 2. Values of n_s and r with respect to varying values of ζ and V_3.

ζ	n_s	r
1	0.9641	0.000011
1.0001	0.95490	0.0006088
1.0005	0.91746	0.00887
1.00001	0.96320	0.02412
2	2×10^{-15}	5.9084

ζ	n_s	r
9900	0.9775	0.0206
10000	0.96412	0.0207
10010	0.96279	0.02072
10100	0.95079	0.02081
11000	0.83318	0.02171

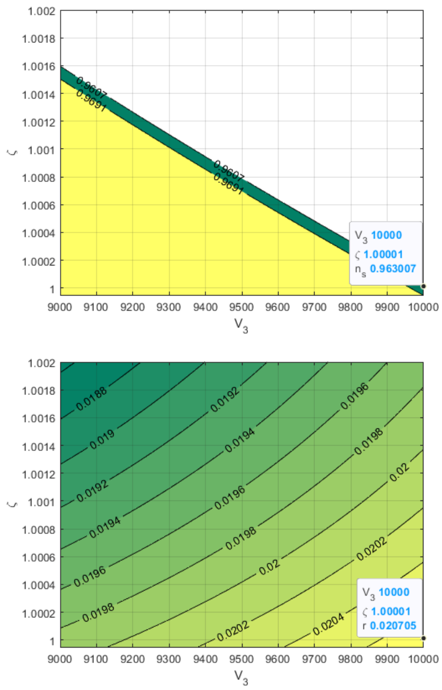


Fig. 5. The accepted value of n_s (left) in green shaded region based on Planck collab. 2018 [27] and accepted values of r (right) with respect to alteration of free parameters V_3 and ζ while preserving other parameters.

RECTIFICATION OF MODELS ASSOCIATED TO APPROXIMATION

We have mentioned earlier that the approximations that have been conducted in the theory need to be checked. According to previous works (see refs. [14, 24, 12, 13]), this has to be done in order to determine the vibility of the proposed theory. In the power law model we checked that there are several approximations do not hold true. For instance, $\ddot{f} \sim \mathcal{O}(10^{-2})$ which has value 0.0149 is in the same order with $H\dot{\phi} \sim \mathcal{O}(10^{-2})$ and is greater than it with value 0.0145. So does for $H^4 f' \sim \mathcal{O}(10^3)$ is the same order with $V' \sim \mathcal{O}(10^5)$ and the last approximations which conducted in the differential equations of potensial also does not hold true where $2\xi \sim \mathcal{O}(10^{-5})$ while $f'' \sim \mathcal{O}(10^{-5})$ hence they are in the same order.

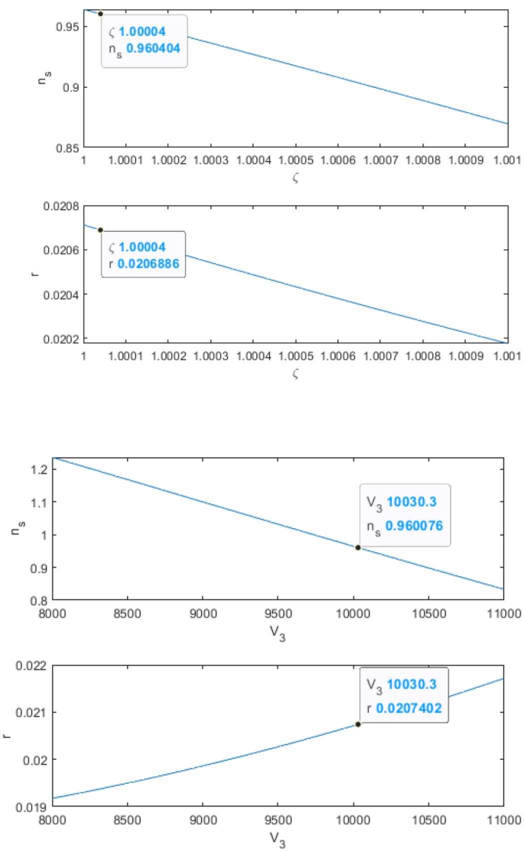


Fig. 6. Plot of n_s and r over varying (a) ζ within range [1, 1.001] and (b) V_3 within range [8000, 11000]. We see here that these parameters controlling the values of n_s and r conveniently. While ζ give a nearly linearly decreasing for both n_s and r, on the other hand V_3 acts differently on ns and r, respectively. As can be seen the latter give a nearly linearly increasing value of r.

Therefore, these approaches rendered invalid. However, to avoid this incompatibility, we have carried out cancellation of preferred approximation. Here, we choose to not approx $\ddot{f} \ll H\dot{\phi}$. Consequently, Hubble derivative equation in eq. (8) is modified where aforementioned aproximations is no longer imposed. We choose to cancel this approximation because is seems to be the simplest one, where we don't have to modify all of equations in theory. Notice that if we modify only this Hubble derivative equation, it will affect ϵ in eq. (61) to form

$$\epsilon = \frac{1}{2(1-\zeta\phi)}\left[\frac{f'^2}{f''^2}(1+f''H^2) - (\zeta\phi - Hf')\frac{f'}{f''}\right] \qquad (90)$$

and no need to modify other equation in eq. (8). Subsequently, the resulting free parameters also changed in order to be viable to observation. Furthermore we choose $(\lambda, N, V2, \xi, \zeta, m, c)$ =(0.0001, 60, 1000, 10^{-15}, 0.1, 33, 0) to yield n_s = 0.9644 and r = −0.045 which reconcilable with observation. The resulting tensor spectral index is n_T = 0.002 which is nearly scale invariant. The slow-roll indices also hold expected result, where $(\epsilon, \eta, \delta_{Q_S}, \delta_{c_S}, \delta_{Q_T}, \delta_{Q_T})$ = (-0.000266, 0.0315, 0.0635, -0.0091, -0.015, 1.06×10^{-10}) have very small values. Finally, the other approximations hold true when using this new approach where $\dot{H} \sim \mathcal{O}(10^2) \ll H^2 \sim \mathcal{O}(10^5), \dot{\phi} \sim \mathcal{O}(10^2) \ll V \sim \mathcal{O}(10^6), \ddot{\phi} \sim \mathcal{O}(10^2) \ll H\dot{\phi} \sim \mathcal{O}(10^4), \ddot{F} \sim \mathcal{O}(10^1) \ll H\dot{F} \sim \mathcal{O}(10^2), \ddot{E} \sim \mathcal{O}(10^{-13}) \ll H\dot{E} \sim \mathcal{O}(10^{-11}), \dot{H}\dot{f} \sim \mathcal{O}(10^{-9}) \ll H^2\dot{f} \sim \mathcal{O}(10^{-6}), H^3 f'\dot{\phi} \sim \mathcal{O}(10^{-3}) \ll V \sim \mathcal{O}(10^6), H^4 f' \sim \mathcal{O}(10^{-1}) \ll V' \sim \mathcal{O}(10^5), H^3 E'\dot{\phi} \sim \mathcal{O}(10^{-5}) \ll V' \sim \mathcal{O}(10^5)$, and $2\xi \sim \mathcal{O}(10^{-15}) \ll f'' \sim \mathcal{O}(10^{-11})$ are all hold true.

In exponential models, several approximations also do not hold true. For instance $\ddot{F}$ has the same order as $H\dot{F}$, $\ddot{f}$ has the same order to $H\dot{f}$ like in power law case, and $H^4 f' \ll V'$ also does not hold true. In this case, we also apply the same manners as in the power law case where Hubble derivative equation is modified do to cancellation of approximation $\ddot{f} \ll H\dot{f}$. For this model we choose these example of free parameters $(V_1, N, \zeta, \xi, m, c)$ = (10000, 60, 0.1, 10^{-10} , 350, -330) to yield n_s = 0.9649 and r = 0.016 in order to be compatible with observation values in eq. (60). The resulting tensor spectral index is $n_T = -0.000082$ which is nearly scale invariant. The slow-roll indices results are also as expected where $(\epsilon, \eta, \delta_{Q_s}, \delta_{c_s}, \delta_{Q_T}, \delta_{Q_T})$=(0.000014, -0.000014, -0.000022, 0.00117, 0.000040, 0). Finally, the approximations have examined and rendered all valid, where $\dot{H} \sim \mathcal{O}(10^{-145}) \ll H^2 \sim \mathcal{O}(10^{-140}), \dot{\phi} \sim \mathcal{O}(10^{-146}) \ll V \sim \mathcal{O}(10^{-140}), \ddot{\phi} \sim \mathcal{O}(10^{-148}) \ll H\dot{\phi} \sim \mathcal{O}(10^{-143}), \ddot{F} \sim \mathcal{O}(10^{-146}) \ll H\dot{F} \sim \mathcal{O}(10^{-145}), \ddot{E} \sim \mathcal{O}(10^{-158}) \ll H\dot{E} \sim \mathcal{O}(10^{-153}), \dot{H}\dot{f} \sim \mathcal{O}(10^{-226}) \ll H^2\dot{f} \sim \mathcal{O}(10^{-221}), H^3 f'\dot{\phi} \sim \mathcal{O}(10^{-291}) \ll V \sim \mathcal{O}(10^{-140}), H^4 f' \sim \mathcal{O}(10^{-288}) \ll V' \sim \mathcal{O}(10^{-141}), H^3 E'\dot{\phi} \sim \mathcal{O}(10^{-293}) \ll V' \sim \mathcal{O}(10^{-141}), and\ 2\xi \sim \mathcal{O}(10^{-10}) \ll f'' \sim \mathcal{O}(10-6)$ are all hold true.

ON THE VIOLATION OF NULL ENERGY CONDITION (NEC)

In the standard single-field slow-roll model of inflation, the power spectrum for the tensor perturbation is parametrized by a power-law [28]

$$\mathcal{P}_T^{vac} = A_T \left(\frac{k}{k_0}\right)^{n_T} , \tag{91}$$

where AT is the amplitude. It is showed that on the superhorizon scale, $k \ll k_0 = aH$, the power spectrum is almost scale invariant that all the GW productions have all the same amplitude [21]. This means that almost all gravitational waves produced at that scale are ”frozen”. In addition, standard single-field slow-roll model possess

$$n_T = -2\epsilon \tag{92}$$

then the tensor spectral index must be negative, $n_T < 0$, so that $\dot{H} \ll 0$ to satisfy Null Energy Condition (NEC) [29]. In this case, the spectrum is named red, whereas if $n_T > 0$ is called blue and $n_T = 0$ is called scale invariant [21]. The standard inflation theory, namely slow-roll inflation which only has a scalar field term and Einstein-Hilbert only produces a red spectrum $n_T < 0$, which can be seen from the eq. (92). Since $\epsilon > 0$ for $\dot{H} \ll 0$, does not arise from this theory [21, 30, 23]. On the other hand, Horndeski’s theory has the advantage that the resulting spectral index allows the blue spectral [21, 30], without NEC violations, in contrast to the case of the standard slow-roll mentioned earlier [21]. This can be done by applying $2\epsilon + \delta_{Q_T} + \delta_{c_T} < 0$ to the eq. (51) but still retaining the condition $\epsilon > 0$ which also means $H < 0$ according to the inflation scenario, so this does not violate the NEC, see also ref. [21].

Then, our result in section 6, the rectified power law model is violating NEC when using corresponding given free parameters value since they produce negative . We can search another region in which accepted observational quantities are laying. We found that altering some free parameters’ values leads to non-violated NEC regions. For example if $(\lambda, N, V2, \xi, \zeta, m, c)$ = (0.0001, 60, 1000, 10^{-15}, -0.1, 16.5, 0), the obtained observational indices are $n_s = 0.9649$ dan $r = 0.0019$ which are accepted to latest Planck data [27], and tensor spectral index is $n_T = -0.0000023635$ which is nearly scale invariant and has red spectrum. As can be seen here, we obtain positive , hence it does not violating NEC. Notice that the only difference of these free parameters with the previous one in sec. 6 is the ζ value. We only alter its sign while preserving the other values and we get the new region which is acceptable to observation and does not violating NEC. This is an interesting feature because there might be significant characteristic about this non-minimal coupling term

This discussion raises the question ”why is the blue spectral index?”. Gravitational waves are thought to be the source of the CMB-mode polarization on a large scale [31, 23], thus if this mode is detected then the spectral index of gravitational waves can be obtained. This mode is very weak on a large scale, while on a small scale it could appear from the gravitational lens [23]. However, the spectral index nT can be positive (blue) in some ranges as observed in the BB BICEP2 spectrum [23, 31].

Up until now, the detection of primordial gravitational waves has been difficult. Due to the weak gravitational interaction, gravitational waves cannot interact with the surrounding media, thus gravitational waves decoupled and propagated freely in the universe after they were produced until now [33]. In contrast to the CMB which releases at energy about 0.3 eV, the primordial gravitational waves are expected to escape at a very high energy scale (approximately the Planck scale) [33]. In addition, since the amplitude is very small, the presence of astrophysical objects, galactic dust and gravitational lensing make this signal even more difficult to be detected [33]. Currently, there is no detection of the primordial gravitational wave signal [33, 24, 30, 21].

According to [36], the challenge for experimental researchers is not only to detect the spectrum of gravitational waves via B-mode polarization but also from the spectral slope. If the result gives a blue spectrum, the inflation theory which gives a red spectrum can be excluded. The solutions are whether to use another alternative theory, namely the superstring theory which always produces a blue spectrum [36] or to use the inflation theory which allows the emergence of a blue spectrum index, for example Horndeski's theory that have been used in this study. The search for an inflation model that produces a primordial tensor fluctuation power spectrum with blue tilt and is consistent with the latest cosmological observations turns out to be an interesting thing [30]. Some work on this subject is can be seen in refs. [36, 30, 21, 23].

For the future research, we could dig deeper about the obtained sign of for each model and associating it to energy scale. We also can check the option to consider effective value of e-fold which can be checked in refs. [21, 32]. The blue or red tensor spectral is also become an interesting consideration. The NEC violation in some region is appealing to be pondered further, whether there exist some special features in the preferred regime. For example its relation to signature of quantum gravity (see, e.g. [39]). Futher review about this topic can be checked in refs. [37, 38, 39]. Additionally, we can check about the non-gaussianities in the model we've proposed.

CONCLUSION

In conjunction with GW170817 event, some of modified gravity (MG) theories were disqualified including Einstein-Gauss-Bonnet theory. We added two more couplings to the action, NMC and NMDC, with some considerations as we have discussed in section 1 and we have showed in this work, the theory still can be manageable to fit in. Furthermore, we exploit Horndeski theory when considering perturbations dynamics. Then we obtained the constraint eq. (55) after applying $c_T = 1$.

Two models of Gauss-Bonnet scalar coupling function has been evaluated in this framework within slow-roll approximations. Then we got equations for potential by solving differential eq. (58). In these models, $f'' \gg \xi$ approach was applied to avoid non linearity and turned out not valid on power law model. Some approximations, including the aforementioned one, that have been conducted do not hold true. Hence we rectified the models by no longer applying $\ddot{f}\xi \ll H\dot{f}$ in the background equation of motion. Consequently, Hubble derivative equation in eq. (8) was altered and also changed equation for . Then we repeated the process to find the suitable free parameters in order to be compatible to the latest Planck 2018 observation.

As we can see in section 5, each form of scalar function affects the free parameters differently thus the behaviour of each dynamic acts peculiarly in resulting observational indices. Nevertheless we obtained the regimes which coincides with expected observational value of ns in green shaded region based on eq. (60) as can be seen in figures [1-5]. We searched aforementioned regimes by varying several free parameters. In the end some free parameters are being in control to change observational indices' values conveniently, while some others could result poor value of observational indices as can be seen in section 5. This is an interesting fact to be reviewed more in the future. We can dig deeper about how important these coupling constant to create some features in inflationary scenario. Nevertheless, since we searched the accepted region ”manually” hence followed by the parameters, these controlling parameters are model-based and only give us the local solutions. In the future, we will search another sophisticated method to obtain more precise values and visualization.

We also discovered that the rectified power law model is in violation of the NEC where the obtained is negative. Then we try to find another region with no NEC violations. As a result, both scenarios are provided.

Furthermore, in terms of tensor spectral index, we obtain a nearly scale invariant value as expected since the RHS of eq. (51) were approximated small. We obtain blue spectrum tensor spectral index, $n_T > 0$, in rectified power law model but at the same time violating NEC. The rest of the models have a red spectrum and at the same time do not violating NEC since $\epsilon > 0$.

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