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# Numerical Fit to the Ashkin Experiment for the 4-Different Formulas of Volume Momentum Proposed by Abraham, Minkowski, Peierls, and Klima-Petrzilka

*Sidikrubadi Pramudito and Yuliati Herbani*
*Laboratory of Theoretical and Computational Physics*
*Department of Physics, Faculty of Mathematics and Natural Sciences*
*Bogor Agricultural University*
*E-mail : sdpramudito@yahoo.com*

***Abstract***
*The momentum flux density of electromagnetic radiation in dielectric media (Abraham-Minkowski problem) has not yet been solved while this question seems to be settled for collisionless fully ionized plasma. Therefore, the experiments of Ashkin are analyzed for a numerical evaluation of the total momentum transferred by laser beam to the sphere suspended in a fluid using four competing formulas for volume momentum, given by Abraham, Minkowski, Peierls, and Klima-Petrzilka. It turns out that at least for this condition of high frequency radiation the Minkowski's formula as predicted by Gordon arrives at the best fit with experiments and no term of Claussius-Mosotti media are involved at this process of radiation forces.*

***Keywords****: radiation forces, dielectric, laser.*

***Abstrak***
*Ekspresi rapat fluks momentum gelombang elektromagnetik untuk medium plasma yang terionisasi sempurna telah mencapai bentuk final yang eksak, namun tidak untuk medium dielektrik. Oleh karena itu, eksperimen Ashkin akan dianalisis dalam kerangka evaluasi numerik untuk menghitung total momentum yang ditransfer oleh berkas laser kepada sebuah bola dielektrik yang terendam di dalam suatu fluida dengan menggunakan empat ekspresi volum momentum yang dikemukakan oleh Abraham, Minkowski, Peierls, dan Klima-Petrzilka. Hasilnya menunjukkan bahwa formula Minkowski - sebagaimana yang telah diprediksikan oleh Gordon untuk radiasi frekuensi tinggi – yang paling cocok dengan eksperimen Ashkin dan tidak ada suku Claussius-Mosotti yang terlibat dalam proses tersebut.*

***Kata kunci:*** *gaya radiasi, dielektrik, laser.*

## 1. Introduction

The Abraham-Minkowski problem of the correct form of momentum of light in a refractive media has not been solved yet. In connection with this problem, many articles have been written, for instance by Klima and Petrzilka[1] and by Peielrs[2], both in 1975 which have the same models except the correction factor, $\sigma$, which was equal to 1/3 for the former and 1/5 for the latter.

The discussion of the Abraham-Minkowski problem may arrived at a solution at least for collisionless plasma[3,4] based on the fact that the final formula of the nonlinear force (generally containing ponderomotive and nonponderomotive terms) could be proved from the momentum conservation of laser-plasma interaction[3] for non-transient condition while the transient case after several disputes was arriving at a formulation[5] which was proved to be Lorentz and gauge invariant[6]. However, the problem seems to be still open for the case of dielectric or media that not fully ionized. The following detailed numerical calculation of the forces of the laser radiation to uniform dielectric sphere embedded in homogeneous fluids permits a clear distinction between the different models. The comparison of these results with measured velocities in the experiments by Ashkin[7] permits a clear decision about the suitable theoretical model. This may provide another access to the solution for the Abraham-Minkowski problem for dielectric materials.

## 2. Method of Calculation

First we are summarizing different formulas for the volume momentum of the electromagnetic wave in media:

$$\mathbf{g} = \frac{1}{c^2}\mathbf{E}\otimes\mathbf{H} \qquad (1)$$

of Abraham,

$$\mathbf{g} = \frac{n^2}{c^2}\mathbf{E}\otimes\mathbf{H} \qquad (2)$$

of Minkowski,

$$\mathbf{g} = \frac{1}{2}\left[\frac{1}{n} + n - \frac{1}{5}\frac{(n^2-1)^2}{n}\right]\frac{n}{c^2}\mathbf{E}\otimes\mathbf{H}) \quad (3)$$

of Peierls, and

$$\mathbf{g} = \frac{1}{2}\left[\frac{1}{n} + n - \frac{1}{3}\frac{(n^2-1)^2}{n}\right]\frac{n}{c^2}\mathbf{E}\otimes\mathbf{H} \quad (4)$$

of Klima-Petrzilka. $n$ is the optical refractive index, the square is the dielectric constant for non-dissipative media. A further proof of the validity of the result for plasma is that the increase of the momentum of a photon changing from a homogeneous medium into another one needs a compensation just given by the reflectivity of Fresnel formula[8], if no cohesive forces are assumed in fully ionized plasma for the limited[9] plane interfaces the extension of this double layer plasma model to the degenerate electrons of a metal immediately reproduced the experimental value in metal or nuclei[10].

We are using this momentum for calculating the force exchange at the interface of dielectric sphere of refractive index $n_2$ and radius $r_o$, suspended in medium of refractive index $n_1$ under irradiation by a laser beam with wavelength $\lambda$, radius of beam $w_o$ and power $P$, as illustrated in Figure 1. Suppose that the incident wave comes from z-positive to z-negative. The field equations can be written as:

$$E_i = I_0 \exp\left[-\frac{\gamma}{2}(x^2+y^2) - i(kz+\omega t)\right]$$
$$B_i = -\frac{n_1}{c} I_0 \exp\left[-\frac{\gamma}{2}(x^2+y^2) - i(kz+\omega t)\right] \quad (5)$$

whereas $\gamma = w_o^{-2}$

We exclude here the longitudinal components of laser beam[11] since they do not contribute the Poynting terms as shown from the temporal phase shift[12].

Energy flux of the electromagnetic wave defined as:

$$\mathbf{S} = \frac{1}{\mu_o}\mathbf{E}\otimes\mathbf{B} \quad (6)$$

and the momentum flux, or the momentum density times the velocity of light in medium can be written as:

$$\mathbf{f} = C\,\mathbf{S} \quad (7)$$

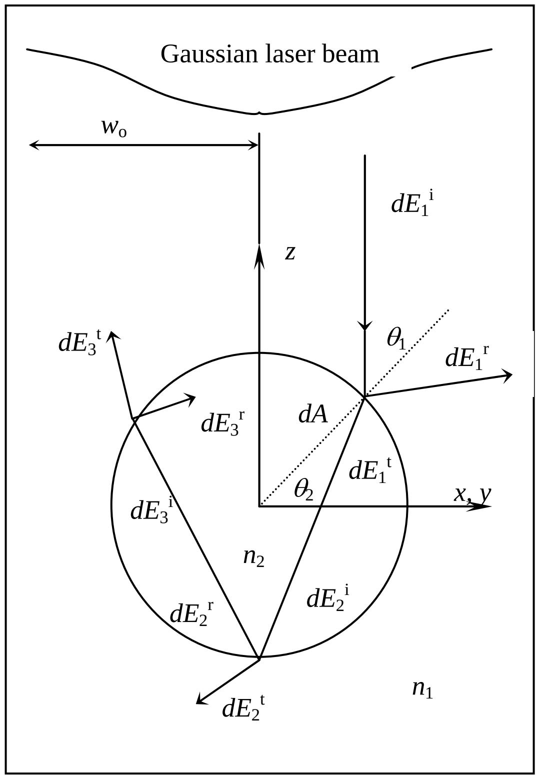


Figure 1. A ray of rate energy $dE_1^i$ hits the top surface of the dielectric sphere at an angle $\theta_1$ and an infinitesimal area $dA$. The sphere has index of refraction $n_2$, and the surround fluid has index of refraction $n_1$. The ray is shown partially refracted and reflected at the lower and the upper surfaces.

where

$$C = \frac{n}{c} \quad \text{for Minkowski}$$
$$= \frac{1}{nc} \quad \text{for Abraham}$$
$$= \frac{1}{2c}\left[\frac{1}{n} + n - \frac{1}{5}\frac{(n^2-1)^2}{n}\right] \quad \text{for Peierls}$$
$$= \frac{1}{2c}\left[\frac{1}{n} + n - \frac{1}{3}\frac{(n^2-1)^2}{n}\right] \quad \text{for Klima - Petrzilka}$$

For incident wave defined in Eq. (5), the energy flux can be written as:

$$S_1^i = \varepsilon_0 n_1 c I_0^2 \exp\left[-\frac{\gamma}{2}(x^2+y^2) - i(kz+\omega t)\right] \quad (8)$$

and its average over a cycle is:

$$\bar{S}_1^i = \frac{1}{2}\varepsilon_0 n_1 c I_0^2 \exp\left[-\frac{\gamma}{2}(x^2+y^2)\right] \quad (9)$$

The relation between the total power of the beam and intensity of the electric field can be determined by integrating the average of energy flux over the cross section of the beam,

$$P = \frac{1}{2}\varepsilon_0 n_1 c I_0^2 \int_0^{2\pi} d\phi \int_0^{w_0} \exp\left[-\frac{\gamma}{2}(x^2+y^2)\right] dr \quad (10)$$

And one finds

$$P = 0.31606 w_0{}^2 \varepsilon_0 n_1 c I_0^2 \quad (11)$$

Now, consider a single ray of rate energy $dE_1^i$ hitting a dielectric sphere at an infinitesimal area $dA$ of the top surface with an angle of incidence $\theta_1$ as seen in Figure 1. The total force on the sphere is the sum of contributions due to the reflected ray and the infinite number of emergent refracted rays of successively decreasing energy is determined by the Fresnel reflectance and transmittance.

By integrating over all beamlet, one arrives at the net force $F$ transferred to the sphere for each of four cases indicated by indices $M$ for Minkowski, A for Abraham, P for Peierls and KP for Klima-Pertzilka. The infinitesimal area dA in spherical coordinate is:

$$dA = r_o^2 \sin\theta \, d\theta \, d\phi \quad (12)$$

Since there is no dependency on $\phi$, Eq. (12) above can be simplified by integrating it over angle $\phi$ to find:

$$dA' = 2\pi r_o^2 \sin\theta \, d\theta \quad (13)$$

Since $x^2 + y^2 = r_o^2 \sin\theta$ the average of energy flux can be written as:

$$\bar{S}_1^i = \frac{1}{2}\varepsilon_0 n_1 c I_0^2 \exp(-\gamma r_0^2 \sin\theta) \quad (14)$$

The rate energy which reaches the infinitesimal area $dA'$ is

$$dE_1^i = \bar{S}_1^i dA' \cos\theta \quad (15)$$

$$dE_1^i = 9.93985 r_o^2 \sin\theta\cos\theta\exp(-\gamma r_o^2 \sin\theta) \quad (16)$$

Following Eq. (7), the rate momentum which comes to the area $dA'$ in $z$-axis can be written as:

$$[df_1^i]_z = -C\, dE_1^i \quad (17)$$

The minus sign shows that this momentum is directed to $z$-negative.

The fraction of reflected and transmitted waves can be determined by using Fresnel formulas in which two cases should be considered. In the first case, the electric field is perpendicular to the plane of incidence and the second case, the electric field is in that plane. The Fresnel formulas for the two cases are given below for a light comes from medium of refractive index $n_1$ to the other of refractive index $n_2$ with $\theta$ and $\theta'$ for an incidence and refractive angle.

1. The electric field at the right angle to the plane of incidence

$$\begin{aligned} E_t &= E_i T_1 \;;\; T_1 = \frac{2n_1\cos\theta}{n_1\cos\theta + n_2\cos\theta'} \\ E_r &= E_i R_1 \;;\; R_1 = \frac{n_1\cos\theta - n_2\cos\theta'}{n_1\cos\theta + n_2\cos\theta'} \end{aligned} \quad (18)$$

2. The electric field is in the plane of incidence

$$\begin{aligned} H_t &= H_i T_2 \;;\; T_2 = \frac{2n_2\cos\theta}{n_2\cos\theta + n_1\cos\theta'} \\ H_r &= H_i R_2 \;;\; R_2 = \frac{n_2\cos\theta - n_1\cos\theta'}{n_2\cos\theta + n_1\cos\theta'} \end{aligned} \quad (19)$$

The rate of energy associated with transmitted and refracted waves, the angle of their propagation relative to the z-axis and the rate momentum in the z-direction due to waves are listed below. For the first penetration, those quantities are:

$$\begin{aligned} dE_1^r &= R_{sq} dE_1^i \quad ; \quad \theta_1^r = 2\theta_1 \\ [df_1^r]_z &= C\, dE_1^r \cos\theta_1^r \\ dE_1^t &= T_{sq} dE_1^i \quad ; \quad \theta_1^t = \theta_1 - \theta_2 + \pi \\ [df_1^t]_z &= C\, dE_1^t \cos\theta_1^t \end{aligned} \quad (20)$$

For further reflection (indicated by m > 1), those quantities are:

$$\begin{aligned} \theta_m^1 &= \theta_1 + m\pi - (2m-1)\theta_2 \\ \theta_m^2 &= \theta_{m-1}^1 + \theta_1 - \theta_2 \\ dE_m^i &= T_{sq} R_{sq}^{m-2} dE_1^i \quad ; \quad \theta_m^i = \theta_{m-1}^1 \quad ; \\ [df_m^i]_z &= C\, dE_m^i \cos\theta_m^i \\ dE_m^r &= T_{sq} R_{sq}^{m-1} dE_1^i \quad ; \quad \theta_m^r = \theta_m^1 \quad ; \\ [df_m^r]_z &= C\, dE_m^r \cos\theta_m^i \\ dE_m^t &= T_{sq}^2 R_{sq}^{m-2} dE_1^i \quad ; \quad \theta_m^t = \theta_m^2 \quad ; \\ [df_,^t]_z &= C\, dE_m^t \cos\theta_m^t \end{aligned} \quad (21)$$

whereas

$$\begin{aligned} R_{sq} &= \frac{1}{2}\left[R_1^2 + R_2^2\right] \\ T_{sq} &= \frac{n_2\cos\theta'}{2n_1\cos\theta}\left[T_1^2 + T_2^2\right] \end{aligned} \quad (22)$$

Table 1. The experimental and calculated radiation forces based on Ashkin experiments[7].

| Quantities | Experiment 1 | Experiment 2 | Experiment 3 |
|---|---|---|---|
| Refractive index of the surround fluid, $n_1$ | 1.33 | 1.33 | 1.33 |
| Refractive index of the sphere, $n_2$ | 1.58 | 1.58 | 1.58 |
| Sphere radius, $r_o$ (μm) | 1.34 | 1.20 | 1.34 |
| Spot size of laser beam, $w_o$ (μm) | 6.20 | 7.50 | 6.20 |
| Wavelength of laser beam, $\lambda$ (μm) | 0.5154 | 0.5154 | 0.5154 |
| Power of laser beam, $P$ (mW) | 19 | 10 | 128 |
| Comparison of resulted forces: | | | |
| $F_{ex}$ (pN)[*)] | 0.6567 | 0.2360 | 5.560 |
| $F_{Ashkin}$ (pN) | 0.7337 | 0.2290 | 4.940 |
| $F_M$ (pN) | 0.7380 | 0.2160 | 4.971 |
| $F_A$ (pN) | 0.4169 | 0.1221 | 2.810 |
| $F_P$ (pN) | 0.5530 | 0.1620 | 3.723 |
| $F_{KP}$ (pN) | 0.5360 | 0.1570 | 3.612 |

[*)] $F_{ex}$ was calculated based on the Stoke's formula in Eq. (24)

For the path of light coming to $dA'$, the change of the rate of momentum gives the force on the sphere. In the z-direction this force is:

$$df_z = \sum_{m=1}^{\infty} \left\{ [df_m^i]_z - [df_m^r]_z - [df_m^t]_z \right\}$$
$$df_z = [df_1^i]_z - [df_1^r]_z - [df_\infty^r]_z - \sum_{m=2}^{\infty} [df_m^t]_z \qquad (23)$$

The total force on the sphere can be found by integrating $df_z$ over the top surface of the sphere (or from $\theta = 0$ to $\theta = \pi/2$)

These calculations are compared with the Ashkin's experiments, in which the maximum velocities of the dielectric spheres under irradiation of $TEM_{00}$ laser beam were measured. By using Stoke's formula,

$$F_{Stoke} = 6\pi\,\eta\, r\, v \qquad (24)$$

the experimental radiation forces can be found. Ashkin himself was using his own empirical formula to find those velocities[7]. The formula is:

$$F_{Ashkin} = \frac{4qr^2 P}{cw_0^3} \qquad (25)$$

where q is the fraction of light effectively reflected back. In this experiment he found $q = 0.062$.

## 3. Result and Discussion

From Table 1., for experiment 1 – 3 it is clear that Minkowski's momentum relation gives the closest result to the experiment (the force is calculated using Stoke's formula). For all experiments, the Peierls and Klima-Petrzilka models give almost the same results as we predicted above and differ strongly in comparison to the experiment. These results are of a good agreement with Minkowski's as predicted by Gordon[13] and supports the view derived by Novak[14]. For very high frequencies (e.g. optical frequencies) the corresponding forces are in accordance with the Minkowski's while experiments for low frequencies[15] agree with Abraham.

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