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An Application of Inversion Technique to 1-Dimensional Gravity Data in Bayesian Framework using Monte Carlo, Metropolis, and Simulated Annealing Algorithm

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Abstract

The purpose of this paper is to present a simulation to the inversion methods applied to geophysical exploration. An application of Monte-Carlo, Metropolis, and Simulated Annealing techniques to 1-Dimensional gravity inversion in Bayesian framework has been studied. Differences between these methods are observed in both single parameter inversion and simultaneous multi parameter inversion. After selecting the best inversion strategy from the three methods, a further investigation was investigated. Multi parameter inversion for two anomalies is simultaneously carried out and results are observed. The synthetical data of GRAV2DC free source were used instead of observed data.

Keywords: Inversion Technique, Monte Carlo, Metropolis, Simulated Annealing, Gravity Method

INTRODUCTION

In any physical system, the input and output parameters are linked by a physical relationship, which may be linear or non-linear. The forward modelling can then be defined in the most general sense as determining an output of a system with known input parameters to the system and known physical relationship. On the other hand, inverse modelling is defined as determining the input parameters by knowing the relationship between input and output and also the output. In yet another class of problems, the physical relation is determined by knowing the input and output to the system. In the equation below the input to the system is m, the output is d and the physical relation is G

$$d = G(m) \tag{1}$$

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In context of geophysical exploration, the inputs to the system are known as model parameters, m and the outputs are known as forward modelled data or synthetic data, d. In case of direct inversion,

$$m = G^{-g}(d) \text{ where } G^{-g} = \left(G^T G\right)^{-1} G^T \tag{2}$$

The operator, G, is a matrix and the model parameters, m and the synthetic data, d are vectors in discrete systems. In linear systems, a linear relation exists between m and d and G is a linear operator. Using direct inversion is not possible in geophysical applications since the matrix G is very large and G^TG is not invertible. A iterative scheme is applied in which the misfit, i.e. residual between the observed data, $dobs$, and the synthetic data, d, is minimized.

Another approach is using the Bayesian framework in which instead of selecting a particular model which minimises the misfit, a probability distribution over the entire model space is obtained. Mathematically,

$$P(m \mid d^{obs}) = \frac{P(m)P(d^{obs|} \mid m)}{P(d^{obs})} \qquad (3)$$

In which the LHS corresponds to the posterior probability distribution function (posterior pdf), $P(m)$ is the apriori pdf. Also if a Gaussian distribution is assumed, $P(d^{obs} | m)$ can be expressed as

$$P(d^{obs|} \mid m) = \exp\left(-\frac{1}{2}\frac{\left\|d^{obs} - d\right\|^2}{\sigma^2}\right) \qquad (4)$$

where $\|.\|^2$ is the L^2 norm and σ^2 is the variance of observed data.

EXPERIMENTAL METHOD

The Monte-Carlo (MC), Metropolis, and Simulated Annealing (SA) methods were applied in these simulations.

Monte Carlo methods are the class of inversion methods in which a model is randomly selected from the model space for computing synthetic data and the posterior pdf. Hence it is also known as random walk. In Markov Chain Monte Carlo methods, the selection of models is such that the posterior pdf is made as close as possible to the desired pdf.

The Metropolis method differs from Monte Carlo in the sense that instead of selecting a model completely randomly, a new model is selected in the vicinity of the existing model and the transition from existing to new model is made with the probability:

$$P(m_{new} \mid m_{old}) = \min \frac{1, P\left(m_{new} \mid d^{obs}\right)}{P\left(m_{old} \mid d^{obs}\right)} \qquad (5)$$

Hence even if the new model gives a larger misfit, it is selected with a probability:

$$\frac{P(m_{new} \mid d^{obs|})}{P(m_{old} \mid d^{obs})} \qquad (6)$$

if this ratio larger tahn a randomly selected number between 0 and 1, the transition is accepted else not. The SA method progressively deforms the shape of the pdf from prior pdf to posterior pdf by decreasing a parameter known as temperature. When the temperature is high (initially), all models are selected, even those with a larger misfit. As the temperature decreases in steps, fewer and fewer models are selected, only those which give a lesser misfit. Hence, it does not get trapped in the local minima like Metropolis. The expression for $P(d^{obs}|m)$ is modified as

$$P(d^{obs|} \mid m) = \exp\left(-\frac{1}{2}\frac{\left\|d^{obs} - d\right\|^2}{\sigma^2 T}\right) \qquad (7)$$

Where T, the temperature is decresed from high (1000) to low (1). (See The Algorithm of SA in the box below)

Algorithm SIMULATED-ANNEALING
Begin
temp = INIT-TEMP;
place = INIT-PLACEMENT;
    while (*temp* > FINAL-TEMP) **do**
    while(*inner_loop_criterion*=FALSE) **do**
        new_place = PERTURB(*place*);
        ΔC =COST(*new_place*) COST(*place*);

        if (ΔC < 0) **then**
            place = new_place;
        else if (ANDOM(0,1) > $e^{-(\Delta C/temp)}$) **then**
            place = new_place;
        temp = SCHEDULE(*temp*);
End.

Figure 1. The algorithm of simulated annealing.

The above methods have been applied to a simple 1D gravity inversion problem. A square anomaly is considered buried in the subsurface. First, only a single parameter, density contrast, is inverted for. A unimodal solution is assumed. Due to lack of true data, a known true density contrast is taken and observed data computed. Then the density contrast is estimated randomly within bound limits. This forms our model space. The results of the three inversion methods are observed for inverting single parameter (density contrast of square anomaly) and then simultaneously for density and width of anomaly.

$$\Delta g = 2G\Delta\rho d\left(\log\frac{x^2 + h_2^2}{x^2 + h_1^2}\right)(6) \qquad (8)$$

We used the square model (6) of gravity modelling in order to build the synthetic data. The G is gravity constant (6.67 x 10^{-3} $Ngr^{-2}m^2$) $\Delta\rho$ is the density contrast, d is the width, and h_1/h_2 is the gap between the lower and upper shape.

SIMULATIONS AND RESULTS

By Using equation (8) we created the forward modelling of synthetic gravity data with random noise as our Observed data.

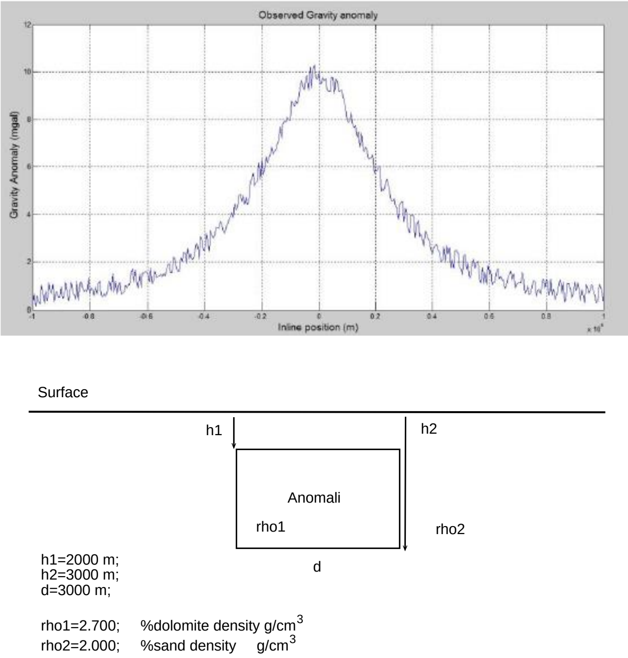
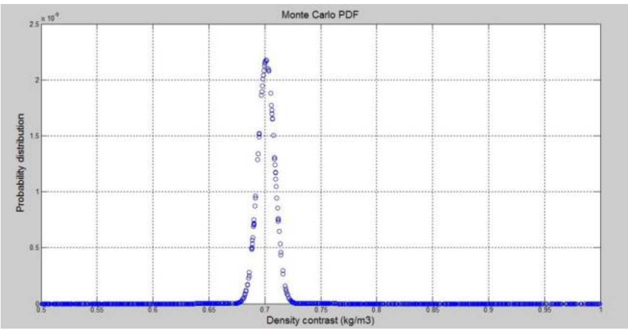
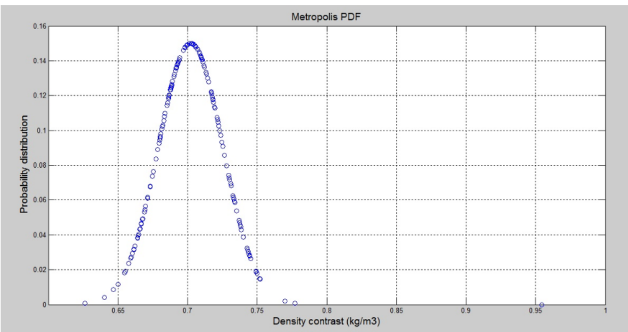


Figure 2. Gravity Anomaly model.

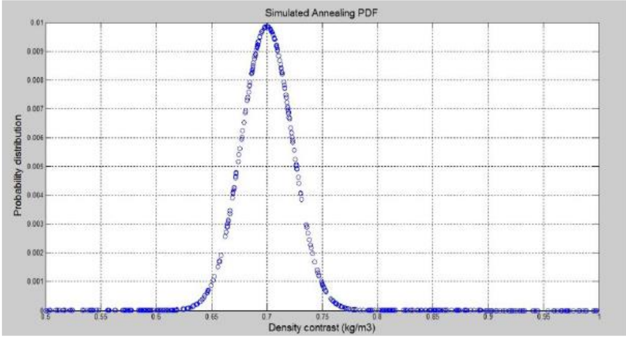
With only single parameter inversion (density contrast), and we set the density contrast of observed model at 0.7 gr/cm^3, then we obtain the probability as it is show in figure 3.



a. Monte Carlo



b. metropolis

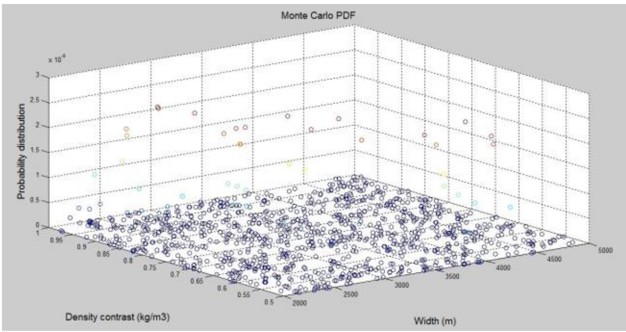


c. Simulated Annealing

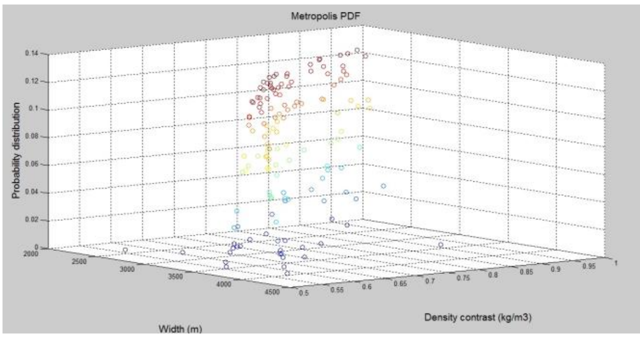
Figure 3. The Posterior pdf obtained by Three Inversion Methods

By performing the Three method, In unimodial solution testing the single inversion parameter, we can see that the posterior pdf has the high probability at 0.65-0.75 gr/cm^3 which is very close to the desired target (0.7 gr/cm^3).

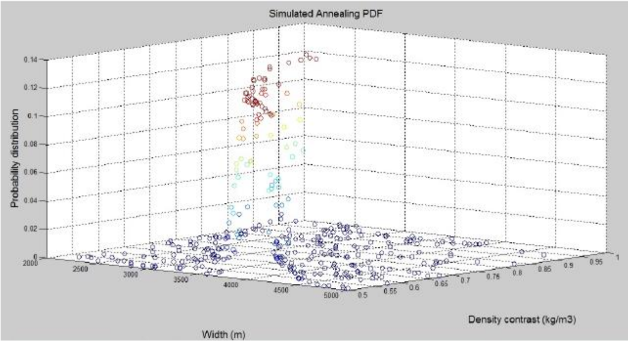
The further simulation of the inversion technique is done by performing simultaneous multi parameter (density contrast and width) inversion: True density contrast and width of the model are 0.7 gr/cm^3 and 4000 m. The result (in figure 4) show us the posterior probability of two parameters inversion in between 3500-4500 m for the width and near 0.7 gr/cm^3 for the density.



a. Monte Carlo



b. Metropolis



c. Metropolis

Figure 4. The Posterior pdf of two parameters inversion

Only the Monte Carlo method are not able to show a good shape of bayesian distribution. The random walk of sampling method in monte carlo might be the main problem in the inversion process.

Multi-Parameter Inversion for two square model

The previous cases utilized a unimodal solution in the density contrast and width of a single anomaly. A single known density contrast and width was used as the real data. In this section, simultaneous inversion for density contrast and width of two square anomalies has been done. The real data that has been used here has been obtained by the freeware GRAV2DC that gives us the gravity anomaly response over a user defined subsurface model. Based on previous results, only simulated annealing has been used for inversion.

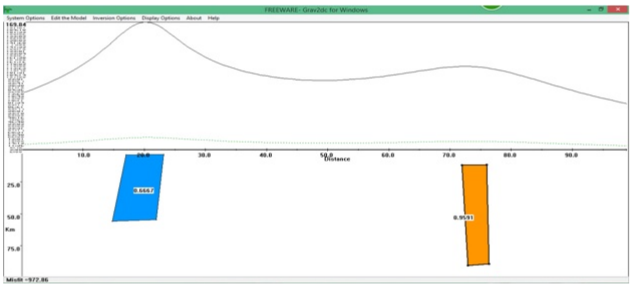


Figure 5. The synthetic data from the GRAV2DC software

There are two models with different shape and density contrast. The main goal of this section is to predict the model in figure 5 with our gravity square model. The data from GRAV2DC software is transferred to MATLAB. In this case, we used 4 parameters as our predictions (the density and width for each anomaly). The parameters for observed data (figure 5) are show in the Table 1.

Table 1. The Parameter's values of simulation anomalies

Anomaly	1(blue)	2(orange)
h1	3000 m	10000 m
h2	55000 m	90000 m
d	7000 m	5000 m
Table of Prediction		
Density	0.5-0.7 gr/cm^3	0.8-1 gr/cm^3
Width	5000-9000 m	3000-7000 m
Temperature	Tmax= 1000 Tmin=1	

We start with the temperature = 1000 and convergence is reached when temperature reaches less than 1. The red line in the figure 7 shows us the best fit model with the observation data after 1000 iterations. This result is obtained when the temperature is equal to 0.043. We get the results of the inversion in figure 6.

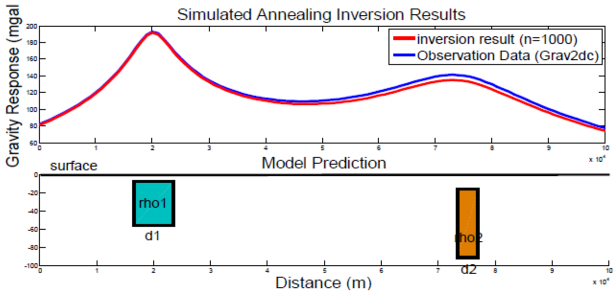
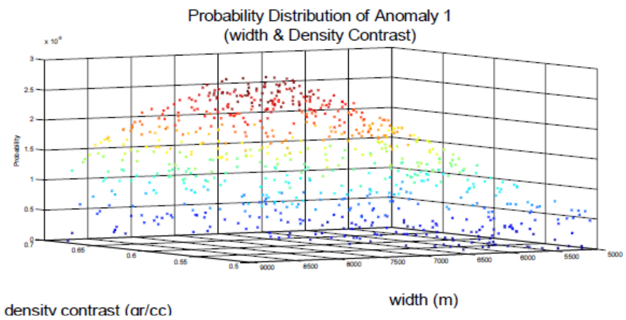
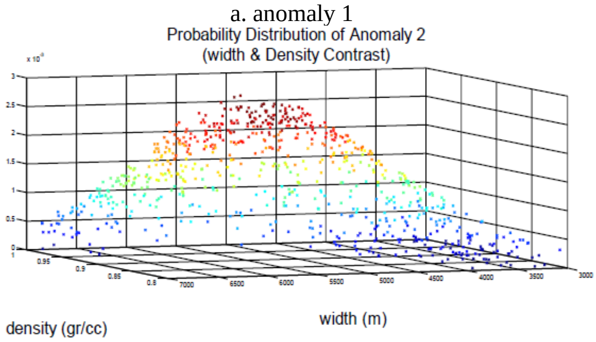


Figure 6. The result of the inversion by SA method.

The probability distribution for the square anomalies are obtained as follows (figure 7).



a. anomaly 1



b. Anomaly 2

Figure 7. The probability distribution of anomalies with respect to the density contrast and width

Ambiguity is observed and no single value of density contrast and width gives the maximum likelihood. The accepted parameter can be seen in the Table 2.

Table 2. The accepted parameter's values of simulation anomalies

Anomaly	1(blue)	2(orange)
Probability	0.9704	
Temperature	0.043	
Density	0.5309 gr/cm^3	0.8825 gr/cm^3
Width	8813 m	5155 m

On comparing the results listed in table 2 and table 1, we can see that our prediction value is good enough since the value of the probability is high (>90%). However, our prediction method is just a simple one. The real models (anomaly 1 and 2) are more complex with their polygonal shape.

DISCUSSION

It can be seen that the model density is high even in regions of very low probability in the Monte-Carlo method. This is a result of completely stochastic sampling of the entire model space. This is improved in the Metropolis method where the model density is high only around the solution. In this case only a unimodal solution has been assumed. However, if there is multimodal solution then several runs of Metropolis with different initial starting models will have to be considered since Metropolis can get trapped in local minima. The MATLAB results from figures 3 – 4 show us that the Simulated Annealing method gives the best results since it accepts models with low likelihoods initially as opposed to the Metropolis method, where the success depends on the accuracy of the initial model. In case of multi-parameter inversion, ambiguity is observed in results. Ideally, a 3D scatter distribution should appear with a maximum at width = 4000 m and density contrast = 0.7 kg/m3. In figure 7, it can be seen that instead of a single value of density contrast and width, different combinations of density contrast and width give maximum likelihoods. Similar observations are made in this case. Hence ambiguity is observed.

CONCLUSION

Three different inversion methods – Monte Carlo, Metropolis and Simulated Annealing - have been tested on 1D gravity inversion problem for a square anomaly. First only a single parameter (density contrast) is inverted for. The probability distribution is observed for the different inversion methods assuming a unimodal solution. Simultaneous multi-parameter inversion (density contrast and width) is also investigated and pdf of different methods is compared. It is concluded that Simulated Annealing gives the best results. In further applications, simulated annealing is used to invert simultaneously for density contrast and width of two square anomalies. The real data is used in this case is derived from GRAV2DC freeware which computes the gravity response over a user-defined subsurface model. Ambiguity is observed in the results.

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