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2-Dimensional Pressure Distribution in Saturated Petroleum Reservoir using Finite Difference

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Abstract

The aim of this project is to solve the Darcy's Equation using the finite difference (FD) method. We test the governing equation by investigating a saturated petroleum reservoir in two-dimensional (2-D) system to describe the distribution of the pressure within the reservoir. We assume that the velocity of the fluid (oil) is incompressible and relatively slow as a consequence that the system is saturated. The model used is a flow in steady state 2-D porous media. We apply the modified form of FN method with Gauss-Seidel to improve the precision of the simulation.

Keywords: reservoir engineering, Porous Media, finite Difference, numerical Simulation

INTRODUCTION

A petroleum reservoir, or oil and gas reservoir, is a subsurface chamber of hydrocarbons contained in porous or fractured rock formations. They are trapped by overlying rock formations with lower permeability. It is important to know the characteristic of the reservoir including the physical and the geometrical changing time by time. Therefore, many scientist and engineer have been conducted research to draw some information about the reservoir behavior. It is popular as called as reservoir engineering.

In this paper, we simulated a forward modeling of the petroleum reservoir based on the fluid flow in porous media. The Darcy's law equation is the main factor in this simulation. Reservoir engineering is based on the understanding of fluid flow in porous media. Because the oil chamber is accumulated in a porous, we must have some data about permeability, porosity, saturation, and relative permeability for oil, for a range of process conditions (Thamir A. Hafedh *et. al.*, 2004). We distinguished the parameters into static parameters (permeability & porosity) and dynamic parameter (pressure). The dynamic parameter is a time dependent variable which is influenced by the static parameters.

Numerical simulation is widely used for predicting reservoir behavior and forecasting its performance. However, the mathematical model used in the simulation requires the knowledge of subsurface properties. Since petroleum reservoirs are relatively inaccessible for sampling, the measurable quantities at the well provide the essential information for reservoir description (Ewing, et. al., 1995). This paper is presented to develop a simulator of numerical modeling for drawing the pressure distribution of the hydraulic head on well injected into the reservoir.

THEORY

Darcy's law is an equation that describes the flow of a fluid through a porous media. The law was formulated by Henry Darcy based on the result of experiment on the flow of water through beds of sand. (Darcy, 1856). It also can be derived from the Navier-stokes equation via homogenization which is analogous to Fourier's law in heat conduction and Fick's law in diffusion.

Darcy's law is a simple proportional relationship between the instantaneous discharge rate through a porous medium, the viscosity of the fluid and the pressure drop over a given distance.

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$$Q = -\frac{kA}{\mu}\nabla P_h \qquad (1)$$

Where Q is the total discharge in unit volume per second (m^3/s) . k is the intrinsic permeability of the medium. A is a unit of area of the medium passed by the fluid (m^2) and ∇P_h is the gradient of hydraulic pressure (Pa/m) and μ is the viscosity dynamic (Pa.s). by dividing both side in the equation (1) with A, therefore

$$q = \frac{Q}{A} = -\frac{k}{\mu}\nabla P_h \qquad (2)$$

Where q is the flux (m/s). however, q is not meant as velocity. Otherwise the fluid velocity v is related to the Darcy flux q divided by the porosity n.

$$v = \frac{q}{n} \qquad (3)$$

by merging the equation (2) and (3), then

$$v = -\frac{k}{n\mu}\nabla P_h = -K\nabla P_h \qquad (4)$$

Where K is Hydarulic Conductivuty. K is also equal to (Daene C. Mckinney)

$$K = \frac{k\rho g}{\mu} = \frac{k}{n\mu} \qquad (5)$$

ρ is density of medium (kg/m^3) and g is gravity constant ($9,8\ m/s^2$) .

There were some **assumption** in this simulation, **first,** This equation are valid for water flow through an aquifer with the value of velocity are sufficiently slow (Singarimbun, 1996) and laminar (low Reynold number). So, the kinetic energy are ignored in the reservoir. **second** we simplified the equation for the Hydraulic Pressure distribution by assuming that the velocity is too slow and limited to zero. We use the 2-D distribution in x & y direction and assume that the pressure distribution does not depend on the z direction (does not change by depth). We ignored the mass and energy balance as a consequence that the reservoir is saturated with no mass transport. **Third**, the dimensional velocity gradient around and within the well are not zero (compressible) but equal to a constant value. However, when the oil still at the reservoir chamber, the gradient is zero (incompressible).

EXPERIMENTAL METHOD

Mathematical Model

In the previous work conducted by Thamir A. Hafedh *et. al.* (2004) yielded that the compressibility of fluid can be quantified by the divergence of the velocity $v_x + v_y$ this measures how much mass enters a small volume in a given unit of time.

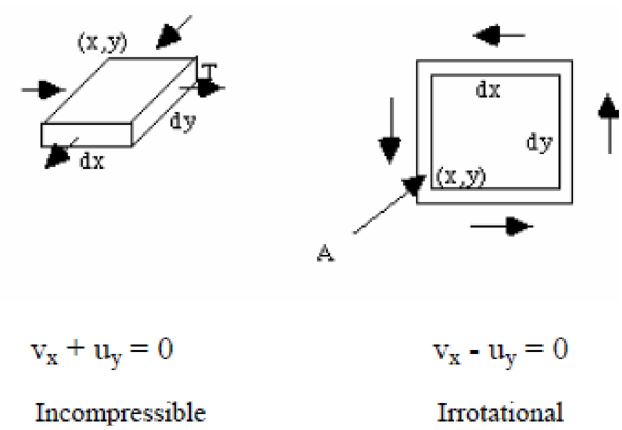

Fig. 1. Incompressible and Irrotational fluid flow

According to Figure 1., the flow in and out of volume is

$$\begin{aligned} dxdyV &= \rho V dy\big(v(x+dx,y) - v(x,y)\big)dt \\ &+ \rho V dx\big(v(x,y+dy) - v(x,y)\big)dt \qquad (6)\end{aligned}$$

if $dxdyV = 0$, the equation (6) represents the incompressible system

$$\rho\left(v_x + v_y\right) = \rho\left(\frac{\partial v}{\partial x} + \frac{\partial v}{\partial y}\right) = 0 \qquad (7)$$

A fluid with no circulation or rotation can be described by the curl of the velocity vector. In 2-D the curl of (u, v) is v_x - v_y. Also the discrete form of this gives some insight to the meaning of this. The circulation or momentum of the loop about the volume ($dx\ dy\ T$) with cross sectional area A and density ρ is

$$\begin{aligned} &\rho A dy\big(v(x+dx,y) - v(x,y)\big)dt \\ &+ \rho A dx\big(v(x,y+dy) - v(x,y)\big)dt \qquad (8)\end{aligned}$$

for irrotational condition ($dxdyV = 0$) the equation (8) become

$$\rho\left(v_x - v_y\right) = \rho\left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right) = 0 \tag{9}$$

For 2-D system of Darcy's law, the equation (3) could become

$$\left(v_x + v_y\right) = \nabla\left(-\frac{k}{n\mu}\left(\frac{\partial P_h}{\partial x} - \frac{\partial P_h}{\partial y}\right)\right) \tag{10}$$

$$\left(v_x + v_y\right) = -\frac{k}{n\mu}\left(\frac{\partial^2 P_h}{\partial x^2} - \frac{\partial^2 P_h}{\partial y^2}\right) \tag{11}$$

According to the second assumption that the velocity gradient of the mass are respectively limited to zero (incompressible and irrotational), so the v_x and v_y are zero. By respecting to the Hydaulic Conductivity *K,* The equation can be expressed as follow

$$-K\left(\frac{\partial^2 P_h}{\partial x^2} - \frac{\partial^2 P_h}{\partial y^2}\right) = 0 \tag{12}$$

The $v_x + v_y$ is the flow rate *R* of the well which have the constant and independent value. This is allow us to set the system as it correspond to the third assumption.

$$-K\left(\frac{\partial^2 P_h}{\partial x^2} - \frac{\partial^2 P_h}{\partial y^2}\right) = R \tag{13}$$

Finally, we get the two equation (12) and (13) of the system that represents the fluid flow in the reservoir.

Finite Difference Approach

The FN method is used to solve the equation by discretization the partial differential equation into a square block with a specific interval. (Desai, 1972). First, we discretize the differencial equation of the system using central difference method.

$$\frac{\partial^2 P}{\partial x^2} = \frac{P_{i+1,j} - 2P_{1,j} + 2P_{i-1,j}}{\Delta x^2} \tag{14}$$

$$\frac{\partial^2 P}{\partial y^2} = \frac{P_{i,j+1} - 2P_{1,j} + 2P_{1,j-1}}{\Delta y^2} \tag{15}$$

By substituting equations (14) and (15) to equations (12) and (13), then

$$-K\left(\frac{P_{i+1,j} - 2P_{1,j} + 2P_{i-1,j}}{\Delta x^2} - \frac{P_{i,j+1} - 2P_{1,j} + 2P_{1,j-1}}{\Delta y^2}\right) = 0 \tag{16}$$

We defined the $\Delta x = \Delta y = 1$, then the equation (12) become

$$P_{1,j} = \frac{P_{i+1,j} + 2P_{i-1,j} + P_{i,j+1} + 2P_{1,j-1}}{4} \tag{17}$$

and for the equation (13)

$$P_{1,j} = \frac{P_{i+1,j} + 2P_{i-1,j} + P_{i,j+1} + 2P_{1,j-1}}{4} + \frac{R}{4K} \tag{18}$$

where *R* is

$$R = \rho\left(v_x - v_y\right) = \rho\left(\frac{\partial v}{\partial x} - \frac{\partial v}{\partial y}\right) \tag{19}$$

For the larger system (in kilometer), it is inefficient to solve the laplace equation by constructing the tri-diagonal matrix. There is another way to solve the laplacian equation using FD method with **Gauss-seidel.**

As we can see in equation (17) or (18), it constructs a diagonal matrix dominantly. A repetition of this approximation will converge to the true solution. By evaluating the equations with the **Liebman'method** we may get

$$P_{i,j}^{new} = P_{i,j}^{*} + \left(1 - \lambda\right) P_{i,j}^{old} \tag{20}$$

Where λ is a relaxation parameter with a value between 1-2, and $P_{i,j}^{old}$ is the initial value of pressure (previous iteration) and $P_{i,j}^{*}$ is the recalculated value of the initial. Therefore we can measure the actual pressure. (Keffer, D,1999)

This process is cycled through for each node at which the solution unknown and then repeated until the solution profile no longer changes (convergence) Or until the error is relatively small.

$$error = \sqrt{\frac{\sum_{i=2}^{n_z} \sum_{j=2}^{n_y} \left(P_{i,j}^{new} - P_{i,j}^{old}\right)^2}{\left(n_x - 1\right)\left(n_y - 1\right)}} \tag{21}$$

We used λ=1,5 and the iterative solution procedure when the error is less than 10^{-4} in maximum 400 iteration.

Model Construction

The first approach, commonly known as geometrical reservoir characterization, focuses on static data (porosity and permeability) . This approach uses the spatial correlation of static data to predict the unknown parameters at unsampled locations (Initial value of Pressure). In the second approach, subsurface properties are estimated through an inverse modeling procedure (Liebman'method), which matches the actual dynamic data (Hydraulic Pressure, P_h).

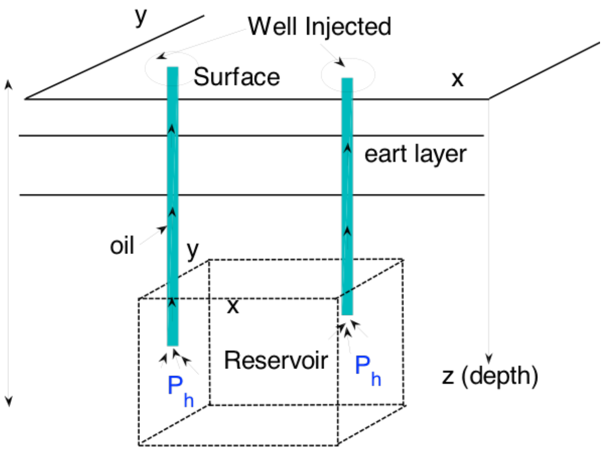

Fig. 2. The Reservoir Model

The Dirichlet Boundary Condition (BC) had been appliaed for the 4 side reservoir wall. We assumed that the pressure for the 4 BC is constant. It means that there is no flow of mass along the wall of reservoir and cause the pressure remains constant.

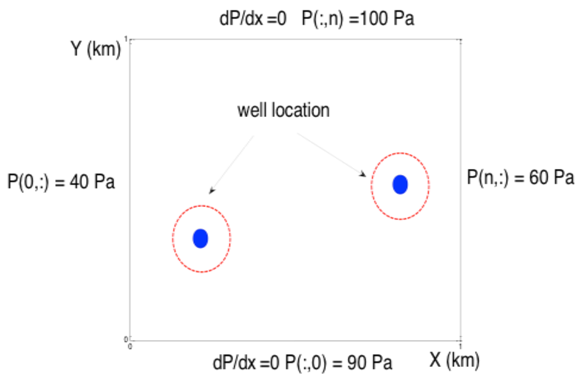

Fig. 3. The Boundary Conditions

We set the physical properties as it is shown in Table 1

Table 1. Physical Parameters

Physical Properties	measured
Intrinsic Permeability (k)	10^{-10} (m^2)
Dynamic Viscosity (μ)	10^{-4} (kg/ms)
Porosity (n)	10%
Hydraulic Conductivity (K)	10^{-4}
Gravity constant (g)	9.8 m/s^2

(x,y)	(1,1) km

RESULTS AND DISCUSSION

First, we draw the pressure distribution in reservoir without attached well. The result is showed by figure 4.

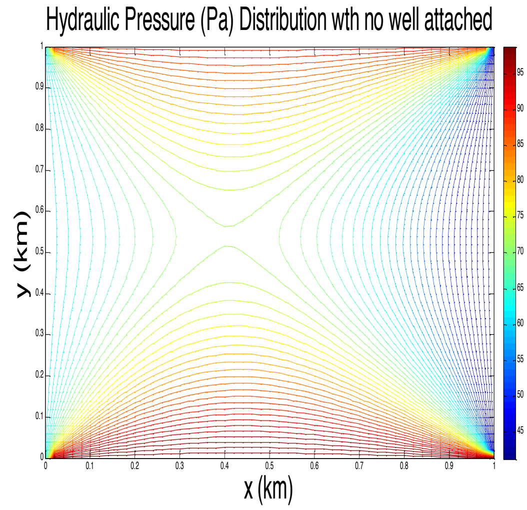

Fig. 4. Pressure Distribution in Reservoir

Where there is no well attached in the reservoir, therefore the fluid flow within it becomes slow and be considered as steady-state fluid flow. It caused the distribution of dimensional velocity is zero too. As we discussed before, we assumed that the reservoir is incompressible and irrotational. The only source are the four side of BC which are have a constant value. They describe that the wall of the reservoir are isolated from the outer side around the system and causing the reservoir becomes saturated.

Reservoir with single well attached

A single well with a flow rate 15 kg/m^{3}s was used in the first numerical experiment. The coordinate of the well is at (x,y) = (0.2 , 0.4) km.

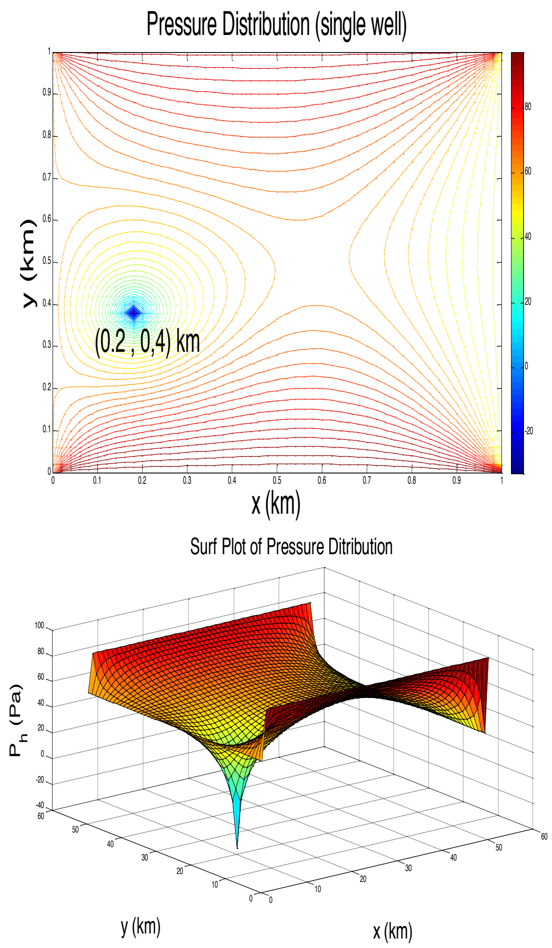

Fig. 5. Pressure Distribution with single well injected at (0.2,0.4) km

What can be inferred from the results is when we attached a well with a small fluid rate (slow) the pressure around the well are no longer incompressible but varied and compressible. It means that the velocity changes around the well cause the gradient of the pressure decrease and become negative. In this case, it indicates that before any steady state condition was achieved, the wells went dry.(lee *et al.,* 1986).

Reservoir with double/triple well attached

For further understanding, we simulate the reservoir with more than one well attached. Here is the table containing the coordinate and the flow rate of each wells

Table 2. Double Wells

Well	Coordinate (x,y) (km)	Flow rate kg/m^3s
Well 1	(0.2 , 0.5) km	8
Well 2	(0.8 , 0.5) km	9

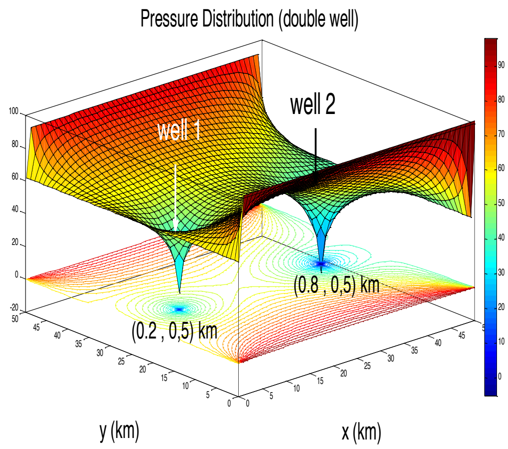

Fig. 6. Pressure Distribution with double well injected

The two well has same K value but different value in flow rate (R) . The surf plot shows us that the gradient of the pressure keep decreasing as well, since the slope are negative. The value of R is responsible in speed of pressure decreasing. The well with less R has slower rate of pressure decreasing.

Table 3. Triple Wells

Well	Coordinate (x,y) (km)	Flow rate kg/m^3s
Well 1	(0.2 , 0.4) km	8
Well 2	(0.8 , 0.2) km	8
Well 3	(0.6 , 0.8) km	9

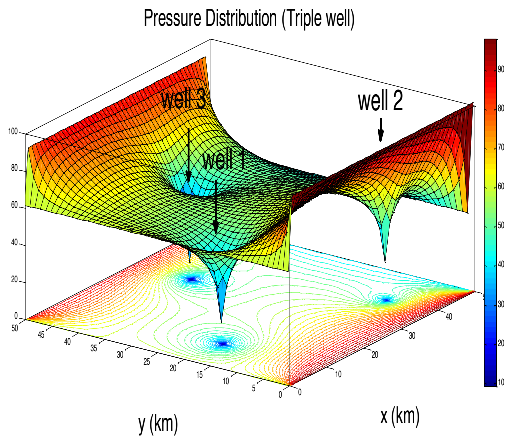

Fig. 7. Pressure Distribution with triple well injected

In triple well, it is similar with the single and double well. The pressure distribution near the well are relatively decrease caused by an incompressible velocity distribution around the well. The higher pressure constant cause the well have more sufficient energy to pump up the oil (well 2) While the others have lower pressure constant causing the rate of the gradient decreasing become narrow.

We have been simulated the Hydraulic Pressure distribution in saturated reservoir with well attached using the Darcy's equation for 2-D steady state flow. There are many assumption which have been applied to the model such as incompressible and irrotational in order to simplify the model. In real application, actually in oil reservoir problems, the soils are usually not fully saturated, and the hydraulic conductivity can be highly nonlinear and can vary with space according to the soil types. Often the soils are very heterogeneous, and the soil properties are unknown. The model could be more complex with irregular shape in 3-D system. However, our simulation has enough assumption to rule out many real application in reservoir engineering.

This simulation can be widely used for many reservoir engineering problem, e.g Geothermic reservoir that installed with double wells (injection well and production well). The difference of this model on comparison with previous model is the flow direction. The production and the injection well has an opposite flow vector. We can simply change the vector sign in the flow rate, and we may see the simulation rsults. However, there must be other treatments since the Geotermic reservoir require two phase fluid flow modeling (steam and liquid) and the Heat transfer phenomenon.

CONCLUSION

The Darcy's law is highly useful in describing the fluid flow in porous media. We have been simulated the Darcy's equation for the saturated reservoir model. We combined the Darcy's equation and the mass balance in two-state. First: incompressible in reservoir, second compressible within the well. In saturated reservoir, the velocity is respectively zero and no outer pressure flow. When the oil are near the well, the pressure gradient are relatively decrease and distribute with slow fluid flow. The different flow rate is responsible in setting the speed of gradient changes. The lower flow rate has slower speed of pressure gradient.

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