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Alternative Model for Quarks and Leptons Using Extra-Dimensional Effective Potential

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Abstract
Assuming that the weak-isospin, particle-type (quark or lepton), color, and generation are internal *quantum numbers, we may generate those quantum numbers by cloning the Dirac operator in four flat extra dimensions and trapping fermions in a harmonic-oscillator potential in the extra dimensions. Particle identification is obtained from the group representation multiplets associated with the choice of the trapping potential in the extra dimensions. The proposed model predicts the existence of* unknown *particles that might be considered as dark matter candidates. The interaction of the particles and the electro-weak bosonic fields which are treated as external fields will be discussed.*

Keywords*: Dirac, flat extra dimension, quark, lepton, harmonic-oscillator.*

Abstrak
Dengan mengasumsikan bahwa weak-isospin*, jenis partikel (*quark *atau* lepton*),* color*, dan generasi adalah bilangan-bilangan kuantum* internal*, kita dapat memperoleh bilangan-bilangan kuantum tersebut dengan menggandakan operator Dirac dalam ruang datar ekstra (tambahan) berdimensi empat dan dengan memerangkap* fermion *dalam suatu potensial getaran harmonik dalam dimensi ekstra. Identifikasi partikel diperoleh melalui multiplet-multiplet representasi grup yang berkaitan dengan pilihan potensial pemerangkap dalam ruang dimensi ekstra. Model yang diajukan meramalkan adanya partikel-partikel tak dikenal yang mungkin dapat dipandang sebagai kandidat materi gelap (*dark matter*). Interaksi partikel-partikel dengan medan* boson electro-weak *yang diperlakukan sebagai medan eksternal akan didiskusikan.*

Kata kunci: *Dirac, dimensi ekstra yang datar, quark, lepton, getaran harmonik.*

1. Introduction

Experimental data show that each quark comes in three different colors, while each lepton is colorless or a singlet in terms of color. Both quarks and leptons can be grouped in several families or generations. The current full width Γ_Z data[1,2] show that it is most likely that there are only three generations for either quarks or leptons. Moreover, both quarks and leptons have weak isospin. We will consider these, the weak isospin, the particle-type (quark or lepton), the color, and the generation as *internal* quantum numbers.

Since there are strong similarities in the electroweak interaction properties from one generation to the others, we will for the most part consider only the first generation of quarks and leptons. This model is proposed as a phenomeno-logical approach for a relatively low range of energy (below 100 TeV) that allow us to ignore the gravitational interaction. By introducing an effective potential in four extra-dimensions that takes the form of harmonic oscillator we will show that we may obtain the internal quantum numbers.

2. Constructing the Field Equation

Assuming that quarks and leptons are really elementary (non-composite) particles, we suspect that the internal quantum numbers such as weak isospin, particle-type (lepton or quark), color, and generation might result from the existence of extra dimensions beyond the ordinary Minkows-ky space-time M^4.

First of all we want to generate weak-isospin. Following the work of Bryan[3], we notice the close resemblance between weak isospin and ordinary or 'mechanical' spin. Since the latter is generated by the Dirac operator $-i\sum_{\mu=0}^{3}\gamma^{\mu}\partial_{\mu} \equiv -i\,\nabla\!\!\!/$ acting in ordinary space-time M^4, we try to clone the Dirac operator to act in the extra dimensions, taken to be Euclidean $\tilde{R}^4$, to yield

$$\left\{-i\sum_{\mu=0}^{3}\left(\gamma^{\mu}\otimes\tilde{1}\right)\frac{\partial}{\partial x^{\mu}}-i\sum_{\tilde{\mu}=1}^{4}\left(1\otimes\tilde{\gamma}^{\tilde{\mu}}\right)\frac{\partial}{\partial\tilde{x}^{\tilde{\mu}}}\right\}\Phi(x,\tilde{x})=0 \quad (1)$$

to generate weak-isospin.

Indeed, the solutions of Eq. (1) exhibit weak-isospin, but with four components – twice as needed. Moreover, we face another problem here, since these solutions are describing a particle that has freedom to move in unlimited distance in the extra dimensions. To avoid this we opt to trap the particle in a potential in the flat extra dimensions. We conjecture that a particular potential might exist which will generate the rest of the internal quantum numbers too.

Neglecting weak-isospin for the moment, we see that the rest of the internal quantum numbers of fermions can result from the $SU(3)\otimes U(1)$ symmetry, where $SU(3)$ corresponds to particle-type and color, and *U(1)* corresponds to generation number. Thus we need to choose a trapping potential whose bound state solutions correspond to certain multiplets of $SU(3)\otimes U(1)$ symmetry.

We recall that solutions of a three-dimensional harmonic-oscillator (HO) potential in the Schrodinger equation can be representations of an *SU(3)* symmetry[3)]. We suspect that a four-dimensional HO potential in the Dirac equation in the extra dimensions will give an $SU(4)$ symmetry which can be decomposed into the proposed $SU(3)\otimes U(1)$ symmetry.

Following the outline above, we can now start constructing the most 'primitive' field equation by introducing a HO potential and cloning the Dirac operator in the extra dimensions to yield the extended Dirac equation:

$$\left\{-i\sum_{\mu=0}^{3}\left(\gamma^{\mu}\otimes\tilde{1}\right)\frac{\partial}{\partial x^{\mu}}-i\sum_{\tilde{\mu}=1}^{4}\left(1\otimes\tilde{\gamma}^{\tilde{\mu}}\right)\frac{\partial}{\partial\tilde{x}^{\tilde{\mu}}}+(1\otimes\tilde{1})\tilde{\alpha}^{3}\tilde{x}^{2}\right\}\Phi(x,\tilde{x})=0 \quad (2)$$

where $\tilde{\alpha}$ is an arbitrary constant, γ^{μ} and $\tilde{\gamma}^{\tilde{\mu}}$ are the Dirac matrices in M^4 and $\tilde{R}^4$, respectively, satisfying anti-commutation relations

$$\begin{aligned}\{\gamma^{\mu},\gamma^{\upsilon}\} &= 2\,g^{\mu\upsilon} \qquad \mu,\upsilon = 0,1,2,3\\ \{\tilde{\gamma}^{\tilde{\mu}},\tilde{\gamma}^{\tilde{\upsilon}}\} &= 2\,\delta^{\tilde{\mu}\tilde{\upsilon}} \qquad \tilde{\mu},\tilde{\upsilon} = 1,2,3,4\,;\end{aligned} \quad (3)$$

with

$g^{\mu\upsilon}$ = diag $(1,-1,-1,-1)$ and $\delta^{\tilde{\mu}\tilde{\upsilon}}$ = diag $(1,1,1,1)$.

If the solutions of Eq. (2) are written as the product of two functions

$$\Phi_{\alpha,\tilde{\alpha}}(x,\tilde{x})=\psi_{\alpha}(x)\otimes\tilde{\psi}_{\tilde{\alpha}}(\tilde{x})\,, \quad (4)$$

then Eq. (2), written in Feynman notation, is separable into

$$(-i\,\nabla+\mathrm{M})\,\psi(x)=0 \quad (5)$$

$$(-i\,\tilde{\nabla}+\tilde{\alpha}^{3}\tilde{x}^{2}-\mathrm{M})\,\tilde{\psi}(\tilde{x})=0\,, \quad (6)$$

where M is the separation constant.

If we identify M as the mass, then Eq. (5) is nothing else than the ordinary Dirac equation. But, unfortunately Eq. (6) does not look like the HO equation when reduced to two-component form (as we will show) and therefore is not likely to give *SU(4)* symmetry. The reason is that basically the Dirac equation has just *SO(4)* symmetry. However, we do not lose hope in obtaining *SU(4)* symmetry since we know that for the right potential, the Pauli reduction of Eq. (6) , like the Schrödinger equation, can exhibit such symmetry[4)]. What we would need is some similar mechanism to act in $\tilde{R}^4$ to suppress either the upper two components or the lower two components of the eigenfunctions. It can be shown that from the remaining components we may build eigenfuctions with *SU(4)* symmetry.

From the application of the Dirac equation to the Hydrogen atom we learn that we can suppress the lower two components of the eigenfunctions to obtain the Pauli equation because of the $\gamma_0 m_0$ term in the equation, provided that $m_0 >> \langle e^2/r\rangle$. We note that the Coulomb potential is unchanged in the reduction.

To imitate the role of $\gamma_0 m_0$ in the hydrogen atom, *i.e.*, to suppress either the lower two components or the upper two components of the eigenfunctions, we introduce in our primitive field equation a phenomenological term $\tilde{\gamma}^5 M_0$, where $\tilde{\gamma}^5$ plays the role of γ_0, and M_0 is very much larger than the effective potential. This will modify our field equation to

$$\left\{-i(\nabla\otimes\tilde{1})-i(1\otimes\tilde{\nabla})+(1\otimes\tilde{1})\tilde{\alpha}^{3}\tilde{x}^{2}+(1\otimes\tilde{\gamma}^{5})M_0\right\}\Phi(x,\tilde{x})=0, \quad (7)$$

which is separable into

$$(-i\,\nabla+M)\,\psi(x)=0 \quad \text{and} \quad (8)$$

$$(-i\,\tilde{\nabla}+\tilde{\alpha}^{3}\tilde{x}^{2}+\tilde{\gamma}^{5}M_0)\,\tilde{\psi}(\tilde{x})=M\,\tilde{\psi}(\tilde{x}). \quad (9)$$

Using a chiral representation of Dirac matrices in the extra dimensions, defining

$$\tilde{D} = \begin{pmatrix}\tilde{\partial}_3-i\tilde{\partial}_4 & \tilde{\partial}_1-i\tilde{\partial}_2\\ \tilde{\partial}_1+i\tilde{\partial}_2 & -\tilde{\partial}_3-i\tilde{\partial}_4\end{pmatrix}, \quad (10)$$

and assuming

$$\tilde{\psi}(\tilde{x}) = \begin{pmatrix}\tilde{\phi}\\ \tilde{\chi}\end{pmatrix}., \quad (11)$$

we can rewrite Eq. (9) as two coupled equations

$$(\tilde{\alpha}^{3}\tilde{x}^{2}+M_0-M)\,\tilde{\phi}+\tilde{D}\tilde{\chi}=0 \quad \text{and} \quad (12)$$

$$\tilde{D}^{+}\tilde{\phi} + (\tilde{\alpha}^3\tilde{x}^2 - M_0 - M)\tilde{\chi} = 0. \qquad (13)$$

Assuming that $M \approx M_0 >> <\tilde{\alpha}^3\tilde{x}^2>$, from Eq. (12) we obtain

$$\tilde{\chi} = \frac{\tilde{D}^{+}\tilde{\phi}}{2M_0} \approx \left(\frac{\tilde{\alpha}}{2M_0}\right)^{\frac{3}{4}}\tilde{\phi}, \qquad (14)$$

which implies $|\tilde{\chi}| << |\tilde{\phi}|$ for $\tilde{\alpha} << M_0$. This means that we have succeeded in suppressing the lower two components of the extra-dimensional eigenfunctions. Substituting Eq. (14) to Eq. (12), and using the identity

$$\tilde{D}\tilde{D}^{+} = -\begin{pmatrix} \tilde{\partial}_1^2 + \tilde{\partial}_2^2 + \tilde{\partial}_3^2 + \tilde{\partial}_4^2 & 0 \\ 0 & \tilde{\partial}_1^2 + \tilde{\partial}_2^2 + \tilde{\partial}_3^2 + \tilde{\partial}_4^2 \end{pmatrix} \equiv -\tilde{\Delta}, \qquad (15)$$

we get the 'Pauli reduction' form of Eq. (9) which to a good approximation has only two-upper-component solutions:

$$\left\{\tilde{\Delta} - 2M_0\tilde{\alpha}^3\tilde{x}^2 + 2M_0(M - M_0)\right\}\tilde{\phi} = 0. \qquad (16)$$

Note, as anticipated, that the HO characteristic of the potential is unchanged. We recognize Eq. (16) as a HO equation in four dimensions with a mass spectrum formula

$$M = M_0 + (2\tilde{N} + 4)\sqrt{\frac{\tilde{\alpha}^3}{2M_0}}; \quad \tilde{N} = 0,1,2,\dots, \qquad (17)$$

where $\tilde{N}$ is the 'total quantum number' analogous to the 'n' in the energy spectrum formula derived from a HO potential in the Schrödinger equation.

Since we are interested in (nearly) massless fermions, wheras M_0 in Eq. (17) is very large, we subtract M_0 from the potential to eliminate the first term of the mass formula. The modified field equation is

$$\left\{-i(\nabla\otimes\tilde{1}) - i(1\otimes\tilde{\nabla}) + (1\otimes\tilde{1})(\tilde{\alpha}^3\tilde{x}^2 - M_0) + (1\otimes\tilde{\gamma}^5)M_0\right\}\Phi(x,\tilde{x}) = 0, \qquad (18)$$

which is still separable into

$$(-i\nabla + M_{\tilde{N}})\,\psi(x) = 0 \quad \text{and} \qquad (19)$$

$$(-i\tilde{\nabla} + \tilde{\alpha}^3\tilde{x}^2 - M_0 + \tilde{\gamma}^5 M_0)\tilde{\psi}(\tilde{x}) = M_{\tilde{N}}\,\tilde{\psi}(\tilde{x}) \qquad (20)$$

where $M_{\tilde{N}}$ is the new separation constant.

In the limit $<\tilde{\alpha}^3\tilde{x}^2> << M_0$ we may write

$$\lim_{\frac{\tilde{\alpha}}{M_0}\to 0}\tilde{\psi} = \begin{pmatrix}\tilde{\phi} \\ \frac{\tilde{D}^{+}\tilde{\phi}}{2M_0}\end{pmatrix} \approx \begin{pmatrix}\tilde{\phi} \\ 0\end{pmatrix}, \qquad (21)$$

where $\tilde{\phi}$ satisfies the HO equation

$$\left\{\tilde{\Delta} - 2M_0\tilde{\alpha}^3\tilde{x}^2 + 2M_0 M_{\tilde{N}}\right\}\tilde{\phi} = 0. \qquad (22)$$

Eq. (22) has a nearly massless mass formula

$$M_{\tilde{N}} = (2\tilde{N} + 4)\sqrt{\frac{\tilde{\alpha}^3}{2M_0}}; \quad \tilde{N} = 0,1,2,\dots \qquad (23)$$

Let us first ignore the spin characteristic of $\tilde{\phi}$ in Eq. (22). The space part of $\tilde{\phi}$ has four-dimensional HO solutions. We will denote these functions as $\tilde{\Phi}$ which have the explicit form

$$\Phi_{\{\tilde{n}_{\tilde{\mu}}\}} = \prod_{\tilde{\mu}=1}^{4} H_{\tilde{n}_{\tilde{\mu}}}(\tilde{\beta}\tilde{x}^{\tilde{\mu}})\,e^{-\frac{1}{2}(\tilde{\beta}\tilde{x})^2}; \quad \tilde{\beta} = (2M_0\tilde{\alpha}^3)^{\frac{1}{4}} \qquad (24)$$

where $\tilde{n}_{\tilde{\mu}}$ = 0, 1, 2, …, $\{\tilde{n}_{\tilde{\mu}}\} = \tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4$, and the $H_{\tilde{n}_{\tilde{\mu}}}$ are Hermite polynomials of degree $\tilde{n}_{\tilde{\mu}}$.

Defining creation and annihilation operators in the extra dimensions, respectively, as

$$\tilde{a}_{\tilde{\mu}}^{+} = \frac{1}{\sqrt{2}}(\tilde{\beta}\tilde{x}_{\tilde{\mu}} - \tilde{\beta}^{-1}\frac{\partial}{\partial\tilde{x}_{\tilde{\mu}}})$$
$$\tilde{a}_{\tilde{\mu}} = \frac{1}{\sqrt{2}}(\tilde{\beta}\tilde{x}_{\tilde{\mu}} + \tilde{\beta}^{-1}\frac{\partial}{\partial\tilde{x}_{\tilde{\mu}}}) \qquad (25)$$

and using the $\tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}}$ as basis functions we can show that the creation and annihilation operators satisfy the commutation relations

$$\left[\tilde{a}_{\tilde{\kappa}}^{+}\tilde{a}_{\tilde{\lambda}}, \tilde{a}_{\tilde{\mu}}^{+}\tilde{a}_{\tilde{\nu}}\right] = \delta_{\tilde{\lambda}\tilde{\mu}}\tilde{a}_{\tilde{\kappa}}^{+}\tilde{a}_{\tilde{\nu}} - \delta_{\tilde{\nu}\tilde{\kappa}}\tilde{a}_{\tilde{\mu}}^{+}\tilde{a}_{\tilde{\lambda}} \qquad (26)$$

which is the algebra of the generators of *U(4)*. Thus $\tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}}$ are basis functions for *U(4)* symmetry.

Furthermore, defining traceless operators

$$\tilde{A}_{\tilde{\mu}\tilde{\nu}} \equiv \tilde{a}_{\tilde{\mu}}^{+}\tilde{a}_{\tilde{\nu}} - \frac{1}{4}\sum_{\tilde{\mu}=1}^{4}\tilde{a}_{\tilde{\mu}}^{+}\tilde{a}_{\tilde{\mu}} \qquad (27)$$

we can show that they satisfy the commutation relations

$$[\tilde{A}_{\tilde{\kappa}\tilde{\lambda}}, \tilde{A}_{\tilde{\mu}\tilde{\nu}}] = \delta_{\tilde{\lambda}\tilde{\mu}}\tilde{A}_{\tilde{\kappa}\tilde{\nu}} - \delta_{\tilde{\nu}\tilde{\kappa}}\tilde{A}_{\tilde{\mu}\tilde{\lambda}}$$
$$[\sum_{\tilde{\mu}}\tilde{a}_{\tilde{\mu}}^{+}\tilde{a}_{\tilde{\mu}}, \tilde{A}_{\tilde{\nu}\tilde{\kappa}}] = 0. \qquad (28)$$

Therefore the $\tilde{A}_{\tilde{\mu}\tilde{\nu}}$ form an invariant subalgebra of *U(4)*, namely, *SU(4)*. Thus the $\tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}}$ are also basis functions for an *SU(4)* symmetry.

Taking the spin part of $\tilde{\phi}$ into account and denoting it as $\tilde{\xi}_{\tilde{m}}$, we can write the explicit extra dimensional part of the solution of Eq. (20) as

$$\lim_{\frac{\tilde{\alpha}}{M_0}\to 0}\tilde{\psi} = \begin{pmatrix}\tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}}\tilde{\xi}_{\tilde{m}} \\ 0\end{pmatrix} \equiv \tilde{\psi}^{(+)}(\tilde{x}) \qquad (29)$$

We will assign the solutions of this form to the doublets of *SU(2)*. The positive sign in Eq. (29) is associated with the positive helicity or the right-handed nature of the solution.

For convenience we will take $\tilde{\xi}_{\tilde{m}}$ to be the eigenvectors of a spin operator $\tilde{S}^3 \equiv \frac{1}{2}\sigma^3$. Then $\tilde{m}_{\tilde{S}} = \pm\frac{1}{2}$ corresponds to

$$\tilde{\xi}_{\frac{1}{2}} = \begin{pmatrix}1\\0\end{pmatrix} \text{ and } \tilde{\xi}_{-\frac{1}{2}} = \begin{pmatrix}0\\1\end{pmatrix} \qquad (30)$$

3. Particle Identification

We can now interpret the previous results. By introducing $\tilde{\gamma}^5 M_0$ term into our field equation, in the limit $\frac{\tilde{\alpha}}{M_0}\to 0$, we manage to suppress the lower two components of eigenfunctions in the extra dimensions. The remaining components consists of a two-component spinor and a four extra dimensional HO soulutions. The first part is responsible for generating the weak isospin, while the second is responsible for generating the rest of the internal quantum numbers.

It has been shown that four-dimensional HO solutions can serve as basis functions for an *SU(4)*-symmetry group. Furthermore, we can decompose this group into $SU(3)\otimes U(1)$ by factoring $\tilde{\Phi}_{\tilde{n}_4}$ out of the $\tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}}$.

We should notice here that since the extra dimensions are real, the four-dimensional HO equation acting in this space only generate the tetrahedral representation of *SU(4)* with lowest irreps 1($\tilde{N}=0$), 4 ($\tilde{N}=1$), 10($\tilde{N}=2$), and so on. The *SU(3)* representations decomposed from this group will only be the 1(n = 0), 3(n = 1), 6(n = 2) and so on.

The weight-diagrams of the lowest lying $SU(2)\otimes SU(4)$ multiplets satisfying Eq. (20), plotted *vs*. quantum number $\tilde{N}$ and weak isospin quantum number $\tilde{m}_{\tilde{S}}$ are shown in Fig. 1.

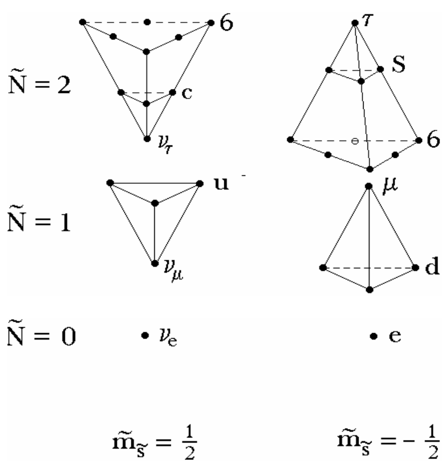

Figure 1 The lowest $SU(2)\otimes SU(4)$ multiplets.

Some particle assignments are indicated. In these diagrams we assume that symmetry breaking has taken place. These diagrams are oriented so that $\tilde{n}_4$-axis points downward for $\tilde{m}_{\tilde{S}} = +\frac{1}{2}$ multiplets and upward for $\tilde{m}_{\tilde{S}} = -\frac{1}{2}$ multiplets, to anticipate the correspondence with the particle masses.

We now come to the particle's identification. First we take the quantum numbers generated from *SU(3)*. The singlets of *SU(3)* would appear to be leptons because no further specification is needed if the weak-isospin and generation numbers are given. The triplets of *SU(3)* would appear to be quarks, since quarks come in three colors. The *SU(3)* sextets and higher apparently represent other kinds of fermions we have not seen. We take the quantum number $\tilde{n}_4$ associated with *U(1)* symmetry to represent the generation quantum number, with $\tilde{n} = 0, 1, 2, \cdots$ corresponding to the first, second, third, … generation. Lastly, we take the quantum number $\tilde{m}_{\tilde{S}}$ associated with *SU(2)* to represent the weak-isospin of the particle. Thus, the complete set of quantum numbers for particle identification is given by $(\tilde{n}_1, \tilde{n}_2, \tilde{n}_3, \tilde{n}_4, \tilde{m}_{\tilde{S}})$

With the identification given above, a set of quantum numbers (0,0,0,0,–½), for example, corresponds to the first generation of leptons with weak-isospin – ½ , *i.e*., the electron. The red up-quark, for example, can be described by the set of quantum numbers (1,0,0,0,½). These particle identifications for the three lowest irreps are listed in Table 1.

What we have done so far, however, has not included chirality. The weak-isospins obtained from Eq. (20) couple with both left-chiral and right-chiral solutions of the ordinary Dirac equation. Since the left-handed and right-handed fermions behave differently in the electroweak interaction, we need to include chirality in our model, *i.e*., link the left-chiral solutions of the ordinary Dirac equation with isospin doublet solutions of the extra dimensions, and the right-chiral solutions of the ordinary Dirac equation with isospin singlet solutions of the extra dimensions. In our model, however, we can not give singlet assignments to right-chiral fermions, *i.e*., our fermions always come out as doublets.

Note, however, that our weak-isospins are four-component spinors, and we have just used the upper two components. If we can find a way to utilize the lower two components, then we will have two independent sets of weak-isospin. Moreover, if they couple with the right-handed and left-handed weak bosons in a correct way, then perhaps we can reproduce the left-right symmetric version of the electroweak interaction proposed by Pati, Salam, and Mohapatra[5-7] prior to symmetry breaking.

Similar approach but with opposite sign of the $\tilde{\gamma}^5 M_0$ term in Eq. (18) will give us solutions that correspond to the negative helicity

$$\lim_{\frac{\tilde{\alpha}}{M_0}\to 0} \tilde{\psi} = \begin{pmatrix} 0 \\ \tilde{\Phi}_{\{\tilde{n}_{\tilde{\mu}}\}} \tilde{\xi}_{\tilde{m}} \end{pmatrix} \equiv \tilde{\psi}^{(-)}(\tilde{x}) \quad (31)$$

Now we have two independent sets of weak isospins. We propose the solutions of the form given in Eq. (29) to couple only with the right-handed bosonic fields and the solutions of the form given in Eq. (31) to couple only with the left-handed bosonic fields. Thus we may assign $\tilde{\psi}^{(+)}(\tilde{x})$ to the doublets of *SU(2)*$_R$, i.e., they represent the right-handed solutions of the extra dimensions, and we may assign $\tilde{\psi}^{(-)}(\tilde{x})$ to the doublets of $SU(2)_L$, i.e., they represent the left-handed solutions of the extra dimensions.

Table 1: Particle Identification

$\tilde{N}$	Quntum number	Particle
0	(0,0,0,0, +½)	ν_e
	(0,0,0,0, –½)	E
1	(0,0,0,1, +½)	ν_μ
	(0,0,0,1, –½)	μ
	(1,0,0,0, +½)	u_{red}
	(1,0,0,0, –½)	d_{red}
	(0,1,0,0, +½)	u_{yel}
	(0,1,0,0, –½)	d_{yel}
	(0,0,1,0, +½)	u_{blu}
	(0,0,1,0, –½)	d_{blu}
2	(0,0,0,2, +½)	ν_τ
	(0,0,0,2, –½)	τ
	(1,0,0,1, +½)	c_{red}
	(1,0,0,1, –½)	s_{red}
	(0,1,0,1, +½)	c_{yel}
	(0,1,0,1, –½)	s_{yel}
	(0,0,1,1, +½)	c_{blu}
	(0,0,1,1, –½)	s_{blu}
	(2,0,0,0, +½)	**“6”**
	(2,0,0,0, –½)	**“6”**
	(0,2,0,0, +½)	**“6”**
	(0,2,0,0, –½)	**“6”**
	(0,0,2,0, +½)	**“6”**
	(0,0,2,0, –½)	**“6”**
	(1,1,0,0, +½)	**“6”**
	(1,1,0,0, –½)	**“6”**
	(1,0,1,0, +½)	**“6”**
	(1,0,1,0, –½)	**“6”**
	(0,1,1,0, +½)	**“6”**
	(0,1,1,0, –½)	**“6”**
…	…	…

The fact that in the limit $p \to E$, the solutions of ordinary Dirac equation, $\psi^{(+)}$ and $\psi^{(-)}$, are the eigenfunctions of γ^5 operator with eigenvalues +1 and −1. The fact that the only difference in the field equation corresponding to the right-handed and left-handed solutions of the extra dimensions is the sign of the $\tilde{\gamma}^5 M_0$ term in Eq. (18) suggests that multiplying that term by γ^5 might result in a desirable relationship between the ordinary- and extra-dimensional solutions. Thus, to account for the chirality into our model, we propose the field equation

$$\begin{aligned}\Big\{ & -i(\nabla\otimes\tilde{1}) - i(1\otimes\tilde{\nabla}) + (1\otimes\tilde{1})(\tilde{\alpha}^3\tilde{x}^2 - M_0) \\ & + (\gamma\otimes\tilde{\gamma}^5)M_0 \Big\}\Phi(x,\tilde{x}) = 0\end{aligned} \quad (32)$$

It can be shown that the general solutions of Eq. (32) in the limit $p \to E$ can be written as

$$\begin{aligned}\Phi(x,\tilde{x}) = {} & c_1\, \psi^{(-)}(x)\otimes\tilde{\psi}^{(-)}(\tilde{x}) \\ & + c_2\, \psi^{(+)}(x)\otimes\tilde{\psi}^{(+)}(\tilde{x})\end{aligned} \quad (33)$$

which links the left-handed solutions of the ordinary Dirac equation only to the left-handed-like solutions from the extra dimensions and the right-handed solutions of the ordinary Dirac equation only to the right-handed-like solutions from the extra dimensions. Fortunately this is consistent with the left and right symmetry as described by Pati, Salam, and Mohapatra model. Thus, after accommodating chirality into our model we may assign fermions to certain multiplets of $SU(2)_L\otimes SU(2)_R\otimes SU(3)\otimes U(1)$, where the $SU(2)_L\otimes SU(2)_R$ corresponds to the left and right weak-isospin, the *SU(3)* corresponds to particle-type and color, and the *U(1)* corresponds to generation number.

However, we still need one final step to modify our field equation. We notice that the four $\gamma^\mu\otimes\tilde{1}$ matrices commute with the four $1\otimes\tilde{\gamma}^{\tilde{\mu}}$ matrices, whereas we would expect all of the gamma-matrices of a true Dirac particle to anticommute. To satisfy this requirement we multiply Eq. (32) on the left by $1\otimes\tilde{\gamma}^5$ to yield a new field equation where all gamma-matrices do anticommute; *i.e.*,

$$\left\{ -i\sum_{u=0}^{7}\Gamma^u \frac{\partial}{\partial z^u} + V + (\gamma^5\otimes\tilde{1})M_0 \right\}\Phi(z) = 0 \quad (34)$$

where

$$\Gamma^u = \begin{cases} \gamma^\mu\otimes\tilde{\gamma}^5 & \dots\ u=\mu=0,1,2,3 \\ 1\otimes\tilde{\gamma}^5\tilde{\gamma}^{\tilde{\mu}} & \dots\ u=\tilde{\mu}+3=4,5,6,7 \end{cases} \quad (35)$$

$$z^u = \begin{cases} x^{\mu} & \ldots\ u = \mu = 0,1,2,3 \\ \tilde{x}^{\tilde{\mu}} & \ldots\ u = \tilde{\mu} + 3 = 4,5,6,7 \end{cases} \quad (36)$$

$$V = (1 \otimes \tilde{\gamma}^5)(\tilde{\alpha}^3 \tilde{x}^2 - M_0). \quad (37)$$

The gamma matrices given in Eq. (35) satisfy the usual anticommutation relations, i.e.,

$$\Gamma^u \Gamma^v + \Gamma^v \Gamma^u = 2 g^{uv} \qquad u, v = 0,1,\ldots,7, \quad (38)$$

where $g^{uv} = diag\,(1, -1, \ldots, -1)$.

4. Discussion

Since the Dirac fields or particles exist in eight dimensions, we assume that the bosonic fields also live in eight dimensions. Here we will deal only with the electroweak bosonic fields.

The electroweak bosonic fields in ordinary space are vector fields. To generalize the electroweak fields in eight dimensions we assume that the electroweak bosonic fields consists of vector-isovector weak-bosonic fields and a vector-isoscalar electromagnetic field.

Since the fermionic fields in our model have left-right symmetry, we assume that after the spontaneous symmetry breaking the electroweak bosons consist of $W_L^{\pm}, W_R^{\pm}, Z_L, Z_R$ and γ (photon). Prior to spontaneous symmetry breaking we will assume that these fields have the form

$$\begin{aligned} \vec{W}_{L\mu}(x,\tilde{x}) &= W_{L\mu}(x) \otimes \tilde{\vec{w}}_L\, e^{-\frac{1}{2}\tilde{\eta}^2 \tilde{x}^2} \\ \vec{W}_{R\mu}(x,\tilde{x}) &= W_{R\mu}(x) \otimes \tilde{\vec{w}}_R\, e^{-\frac{1}{2}\tilde{\eta}^2 \tilde{x}^2} \\ B_{\mu}(x,\tilde{x}) &= B_{\mu}(x) \otimes \tilde{b}\, e^{-\frac{1}{2}\tilde{\eta}^2 \tilde{x}^2} \end{aligned} \quad (39)$$

Demanding that the left-handed fermions interact only with left-handed weak bosons, and similarly that the right-handed fermions interact only with the right-handed weak bosons, while the singlet B boson can interact with either the left-handed or right-handed fermions, we propose that the interaction can be described as

$$\left\{ -i \sum_{u=0}^{7} \Gamma^u \frac{\partial}{\partial z^u} + V + (\gamma^5 \otimes \tilde{1}) \right\} \Psi(z) = -(\gamma^{\mu} \otimes g_0\, \tilde{\vec{\sigma}} \bullet \vec{W}_{\mu} + \gamma^{\mu} g_0{}' B_{\mu})\, \Psi(z) \quad (40)$$

where

$$\tilde{\sigma}^{\tilde{j}} = \begin{pmatrix} \tau^{\tilde{j}} & 0 \\ 0 & \tau^{\tilde{j}} \end{pmatrix}; \qquad \tilde{j} = 1,2,3. \quad (41)$$

with $\tau^{\tilde{j}}$ the 2×2 Pauli matrices acting only on the extra-dimensional space, and

$$g_0 = \begin{pmatrix} g_{0R} & 0 \\ 0 & g_{0L} \end{pmatrix}; \qquad \vec{W}_{\mu} = \begin{pmatrix} \vec{W}_{R\mu} & 0 \\ 0 & \vec{W}_{L\mu} \end{pmatrix}. \quad (42)$$

Assuming that the bosonic fields extend much farther in the extra dimensions than the fermionic fields do, and using the orthogonal property and completeness of the basis functions we may construct the Lagrangian of the interaction[4)]

$$\begin{aligned} \mathcal{L}_0 = \; & \overline{\Psi}_R(z)\gamma^{\mu}(i\partial_{\mu} - \tfrac{1}{2} g_R\, \vec{\tau}\cdot W_{R\mu} - \tfrac{1}{2} g' B_{\mu})\, \Psi_R(z) + \\ & \overline{\Psi}_L(z)\gamma^{\mu}(i\partial_{\mu} - \tfrac{1}{2} g_L\, \vec{\tau}\cdot W_{L\mu} - \tfrac{1}{2} g' B_{\mu})\, \Psi_L(z), \end{aligned} \quad (43)$$

where Ψ stands for any doublet of fermion fields. Eq. (43) holds for any isospin pair of quarks, and any isospin pair of leptons, for any generation. Note that it does not mix quarks with leptons nor particle of one generation with those of another generation.

We note that the Lagrangian derived from our model is different in structure with that of the standard model. In this model, both left-handed and right-handed fermions form isospin doublets, while in the standard model only left-handed fermions form isospin doublets; the right-handed fermions always appear in singlets. While the bosonic fields in the standard model comes automatically from the gauge symmetry, in this model the bosonic fields are taken to be external fields. This results in the disappearing of the kinetic energy terms of the bosonic fields in the Lagrangian.

Ignoring the Cabibbo-Kobayashi-Maskawa mixing, it can be shown that in interacting with weak bosons, the initial fermion may not change generation number, particle type (*e.g.*, from lepton to quark or *vice versa*), color, nor weak isospin. Furthermore, transmuting a left-handed fermion to a right-handed fermion, and *vice versa*, are prevented.

5. Conclusion

An alternative model for quarks and leptons has been proposed by assuming that all particles live in eight-dimensional space-time. The extra four dimensions are assumed to be flat, and most of all particles are trapped in a HO-like potential in the extra dimensions. The spectrum of the solutions to the proposed field equation results in the *internal* quantum numbers that correspond to the weak-isospin, generation, color, and particle-type (quark or lepton). The solution predicts the existence of *unknown* particles that might be associated with the dark matters in the universe. The interaction of the particle with the bosonic (electroweak) fields, treated as external fields, results in the conservation of generation number, particle-type, color, weak-isospin and also chirality. The mass formula, however, is inconsistent with experimental data.

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