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The Constructions of Egg-Shaped Surface Equations using Hugelschaffer's Egg-Shaped Curve

Ahmat Rif'an Maulana[1*], Mahmud Yunus[2], and Dwi Ratna Sulistyaningrum[3]
[1,2,3]Department of Mathematics, Faculty of Mathematics and Natural Sciences
Sepuluh Nopember Institute of Technology
Kampus ITS, Keputih, Sukolilo-Surabaya 60111, Indonesia

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Abstract

Hugelschaffer's egg-shaped curve is egg-shaped curve that is constructed by two non-concentric circles using Newton's transformation known as hyperbolism. This study has goals to construct the egg-shaped surface equations using Hugelschafer's egg-shaped curve that is rotated on *x*-axis, *y*-axis and *z*-axis; to get the volume formula of the egg-shaped solid and the egg-shaped surface area and also to visualize the egg-shaped surface equations using GeoGebra. Hugelschaffer's egg-shaped curve is selected because its equation is simple. The procedures of the construction of the egg-shaped surface equations are done by drawing the curve on *xy*-plane and *xz*-plane, then it is rotated on axes of the coordinate. Whereas, the volume formula of the egg-shaped solid is gotten by using the disk method of the volume integral. The egg-shaped surface area is attained by using the integral of surface area. Visualisation of the egg-shaped surface equations are done by choosing vary of parameter values of the equations that aims to know the effect of the parameter values with the shaped surface.

Keywords: Hugelschaffer's egg-shaped curve, egg-shaped surface equations

INTRODUCTION

This paper is motivated by applications of the egg-shaped curve on egg-shaped gutters in the roof and the ground. It is explained that the cross-section of flowing water is egg-shaped, so they are said to have good drainage. In addition, the egg-shaped have been applicated on speaker design. It is explained that by suppressing various oscillations and echoes occurring inside the speaker, the egg-shaped is used to pursue reproduction of the original sounds. Another application is the egg-shaped sludge digestion chambers. In comparison to previous tubular digestion chamber, it is explained that they are superior in terms of water-tightness and air-tightness [1].

Hugelschaffer's egg-shaped curve is a closed curve that represents the margin line of an egg [2] and often called Hugelschaffer's egg-shaped curve construction [3]. The idea of the construction of Hugelschaffer's egg-shaped curve is constructive procedure of Newton's transformation that is done by two non-concentric circles, known as hyperbolism [4]. The Hugelschaffer's egg-shaped curve has mathematically simple equation, that is identically to an equation of distortion ellips, but the equation is bounded on two dimension [5]. The equation of Hugelschaffer's egg-shaped curve has three shaped parameters. They play important role on the shaped of the egg-shaped. These parameters are *a*, *b* and *w*. If we change the values of the parameter, we can obtain oval egg-shaped, pyriform egg-shaped, circular and elliptical egg-shaped [6].

According to these reasons, in this paper, the equation of Hugelschaffer's egg-shaped curve will be used to construct the egg-shaped surface equation (3D egg-shaped equation) and analyse the effects of the parameter values of the egg-shaped surface equation.

THE EQUATION OF HUGELSCHAFFER'S EGG-SHAPED CURVE

The construction of Hugelschaffer's egg-shaped curve can be obtained from two non-

* Corresponding author.
E-mail address: ahmatrifan@gmail.com

concentric circles using definition of Newton's transformation [7]. That is, we make sketch of circle k_1 with radius a and circle k_2 with radius b, where the center point of circle k_1 and circle k_2 have distance w. It is shown in Figure 1.

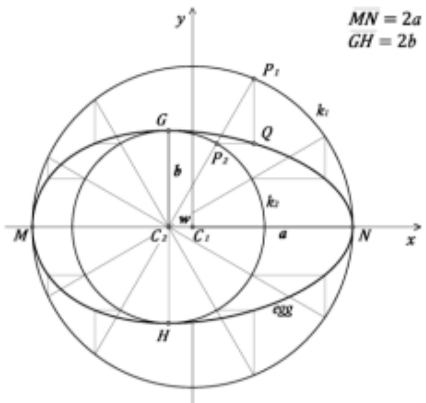

Fig. 1. Construction of Hugelschaffer's egg-shaped curve.

Furthermore, by using the comparison of gradient C_2P_2 and C_2P_1, we obtained the equation of Hügelchaffer's egg-shaped curve, that is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\left(1+\frac{2wx+w^2}{a^2}\right)=1, \quad (1)$$

with $a>b>0$ and $0<|w|<a$; a,b,w constants and MN is major axis of curve and GH is minor axis of curve [7].

RESEARCH METHOD

The finding processes of the egg-shaped surface equations are done by making sketch of the egg-shaped curve on *xy*-plane and *xz*-plane. Then, the curve is rotated on the axes of coordinate. Whereas, the volume formula of egg-shaped solid is attained by using disk method of the volume integral [8]. The egg-shaped surface area is found using integral of surface area that is numerically solved [9]. Visualisation of the egg-shaped surface equation is done by using GeoGebra where the equation must be changed to parametric form.

RESULTS AND DISCUSSION

The construction of the egg-shaped surface equations with x-axis as rotation axis

The construction steps are as follow. *First*, we make sketch of Hügelschaffer's egg-shaped curve (*T* curve) in *xy*-plane where the major axis of the curve is on *x*-axis and the mid-point of the major axis is at the point *O*(0,0,0). It is shown in the Figure 2.

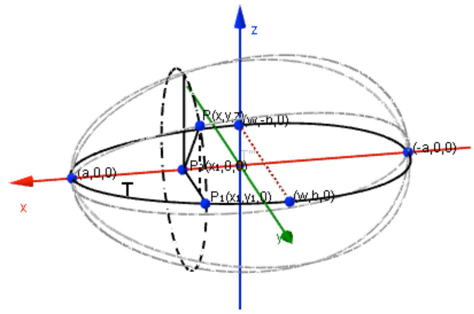

Fig. 2. The *T* curve on *xy*-plane rotated on *x*-axis.

Second, we take any point on *T* curve, let $P_1(x_1,y_1,0)$, and we state the equation of *T* curve at the point P_1. *Third*, the *T* curve is rotated on *x*-axis, so that the point $P_1(x_1,y_1,0)$ forms circle path with point $P_2(x_1,0,0)$ as centre point and radius $|P_2P_1|=y_1$. *Fourth*, we take any point on circle path, let $P(x,y,z)$. Then, we show that the point $P(x,y,z)$ is satisfy the equation of *T* curve in the point P_1, that is by using the relations of radius $|P_2P|$ and $|P_2P_1|$, we project the point $P(x,y,z)$ on *xy*-plane, so that we get the egg-shaped surface equation where the mid-point of major axis is at *O*(0,0,0), that is

$$\frac{x^2}{a^2}+\frac{y^2}{b^2}\left(1+\frac{2wx+w^2}{a^2}\right)+\frac{z^2}{b^2}\left(1+\frac{2wx+w^2}{a^2}\right)=1, \quad (2)$$

where $a>b>0$; $0<|w|<a$; a,b,w constants.

Whereas, the egg-shaped surface equation where the mid-point of the major axis is at $Q(p,q,r)$, that is

$$\frac{(x-p)^2}{a^2}+\frac{(y-q)^2}{b^2}f(x)+\frac{(z-r)^2}{b^2}f(x)=1 \quad (3)$$

where $f(x)=\left(1+\frac{2w(x-p)+w^2}{a^2}\right)$.

The construction of the egg-shaped surface equations with y-axis as rotation axis

The construction steps are similar to the construction of the equation of egg-shaped surface where the *x*-axis as rotation axis. The difference step is only on the first step, that is, we make sketch of *T* curve on *xy*-plane where the major axis curve is on the *y*-axis. It is shown in the Figure 3.

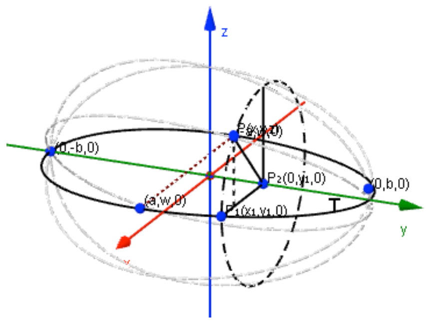

Fig. 3. The *T* curve in *xy*-plane rotated on *y*-axis.

Then, we obtained the equation of egg-shaped surface, where the mid-point of major axis is at $O(0,0,0)$ and the y-axis is as rotation axis, that is,

$$\frac{x^2}{a^2}\left(1+\frac{2wy+w^2}{b^2}\right)+\frac{y^2}{b^2}+\frac{z^2}{a^2}\left(1+\frac{2wy+w^2}{b^2}\right)=1, \qquad (4)$$

where $b>a>0;\ <0|w|<b;\ a,b,w$ constants.

Whereas, the equation of egg-shaped surface where the mid-point of major axis at $Q(p,q,r)$ is

$$\frac{(x-p)^2}{a^2}f(y)+\frac{(y-q)^2}{b^2}+\frac{(z-r)^2}{a^2}f(y)=1 \qquad (5)$$

with $f(y)=\left(1+\dfrac{2w(y-q)+w^2}{b^2}\right)$.

The construction of egg-shaped surface equations with z-axis as rotation axis

If we make sketch *T* curve on *xz*-plane that is shown in the Figure 4.

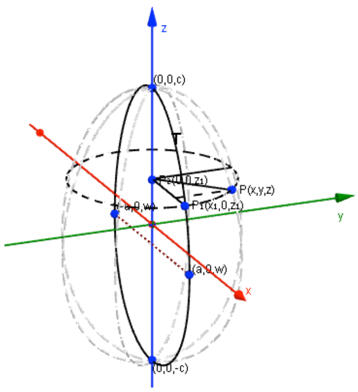

Fig. 4. The *T* curve in *xz*-plane rotated on *z*-axis.

By using the construction steps are similar to previous session, then the equation of egg-shaped surface where the mid-point of major axis is at $O(0,0,0)$ and the z-axis is as rotation axis, is

$$\frac{x^2}{a^2}\left(1+\frac{2wz+w^2}{c^2}\right)+\frac{y^2}{a^2}\left(1+\frac{2wz+w^2}{c^2}\right)+\frac{z^2}{c^2}=1, \qquad (6)$$

where $c>a>0$; $0<|w|<c$; a,c,w constants.

Whereas, the equation of egg-shaped surface where the mid-point of major axis is at $Q(p,q,r)$, is

$$\frac{(x-p)^2}{a^2}f(z)+\frac{(y-q)^2}{a^2}f(z)+\frac{(z-r)^2}{c^2}=1 \qquad (7)$$

where $f(z)=\left(1+\dfrac{2w(z-r)+w^2}{c^2}\right)$.

The volume of egg-shaped solid

The volume of the egg-shaped solid can be obtained by using Hügelschaffer's egg-shaped curve on *xy*-plane that is rotated on *x*-axis. By using the disk method of the volume integral [8], then we obtained the volume formula of the egg-shaped solid, that is

$$V=\pi b^2\left[\left(a^2-\frac{(a^2+w^2)^2}{4w^2}\right)\frac{1}{w}\ln\left(\frac{a+w}{a-w}\right)+\frac{a(a^2+w^2)}{2w^2}\right]. \qquad (8)$$

The egg-shaped surface area

In calculus, to find the surface area of solid that has symmetric axis or is obtained by rotating a curve, we can use formula [8] :

$$A=2\pi\int_a^b f(x)\sqrt{1+(f'(x))^2}\,dx \qquad (9)$$

By stating the Eq. 1 in the form of explicit function $y=f(x)$, and finding the differensial $f'(x)$, then substituted to Eq. (9), we obtained the integrand of the surface area of egg-shaped, that is

$$A=2\pi b\int_{-a}^{a}\frac{\sqrt{(a^2-x^2)(a^2+2wx+w^2)^3+b^2(w+x)^2(wx+a^2)^2}}{(a^2+2wx+w^2)^2}dx \qquad (10)$$

Because the Eq. 10 is difficult to be solved analytically, furthermore the surface area of the egg-shaped (Eq. 10) is calculated numerically.

Visualisation of the egg-shaped surface equations

Visualisation of the egg-shaped surface equations in this paper use GeoGebra software. The Eq. 2 to Eq. 7 are changed in the parametric form :

$$x = a\cos t$$
$$y = \frac{ab\sin t\cos u}{\sqrt{a^2 + 2aw\cos t + w^2}} \quad (11)$$
$$z = \frac{ab\sin t\sin u}{\sqrt{a^2 + 2aw\cos t + w^2}};$$

with $0 \le t \le \pi$ and $0 \le u \le 2\pi$.

For examples, the results of the visualisation of the egg-shaped surface equations (Eq. 2 to Eq. 7) with $a = 2{,}685$, $b = 1{,}835$ and $w = 0{,}45$, respectively are as follow :

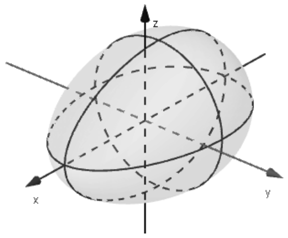

Fig. 5. Egg-shaped surface from Eq. 2.

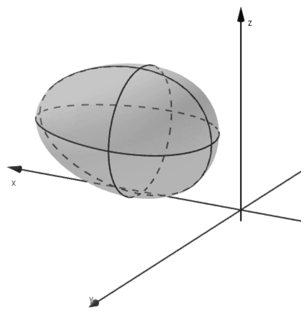

Fig. 6. Egg-shaped surface from Eq. 3 with *Q*(2,3,3).

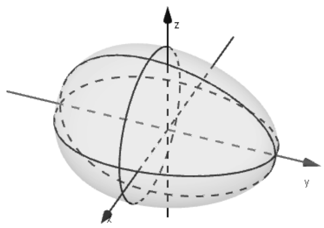

Fig. 7. Egg-shaped surface from Eq. 4.

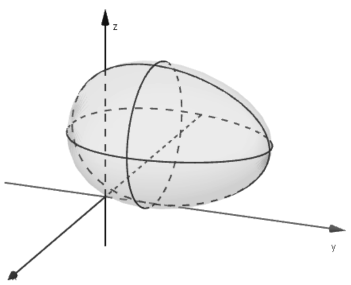

Fig. 8. Egg-shaped surface from Eq. 5 with Q(3,2,3).

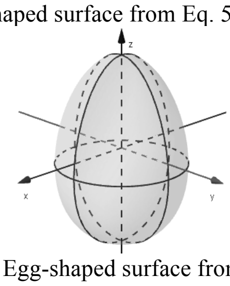

Fig. 9. Egg-shaped surface from Eq. 6.

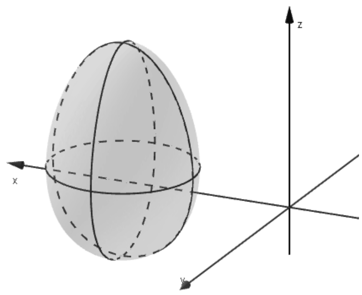

Fig. 10. Egg-shaped surface from Eq. 7 with Q(3,3,2).

If the parameter values of *a*, *b* and *w* are substituted to Eq. 8 and Eq. 10, we obtained the volume is $V \approx 37{,}657$ and surface area is $A \approx 4{,}84$. The corresponding between parameter *a* and *b* and parameter *a* and *w* to the oval egg-shaped surface are $0{,}681 \le \frac{b}{a} \le 0{,}815$ and $0{,}0292 \le \frac{w}{a} \le 0{,}1675$. This interval is measured from 63 chicken eggs. The oval egg-shaped surface can be seen on the Fig. 5 to Fig. 10 where the surfaces is oval or ovoid.

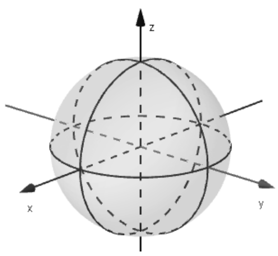

Fig. 11. Sphere with radius 2.

The effect of the parameter values to the egg-shaped surface is as shown on Figure 11 to Figure 14. The Eq. 6 is sphere if we choose $a = b > 0$ and $w = 0$. It is as shown on Fig. 11. Whereas, the Eq. 6 is ellipsoid if we choose $a > b > 0$ and $w = 0$, that is shown on Fig. 12.

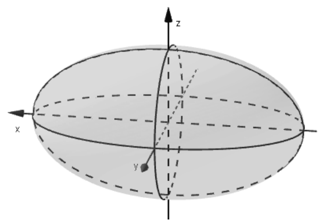

Fig. 12. Ellipsoid.

If we choose the parameter values of *a*, *b* and *w* out of the interval of oval egg-shaped, we can get the surface as shown on Fig. 13. That is, when the value of parameter *a* is more and more large, then the surface does not represent oval egg-shaped.

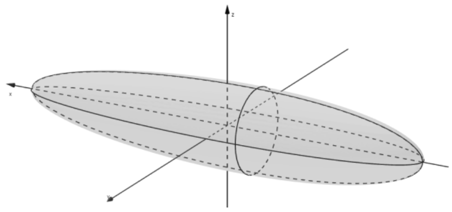

Fig. 13. Surface from Eq. 2 with $b = 0{,}2a$ and $w = 0{,}1047a$.

Meanwhile, if the condition of Eq. 6 is not sufficed, the visualisation of the surface is not egg-shaped. It is as shown on Fig. 14.

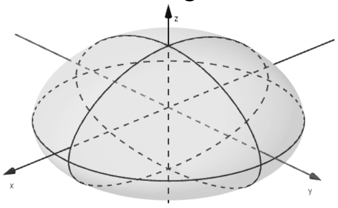

Fig. 14. Surface from Eq. 6 with $a = 3$, $c = 2$ and $w = 0{,}44$.

CONCLUSION

Based on discussion, the effect of the parameter values of the egg-shaped surface equations toward the visualization results are if the values of parameter *a* and *b* are constant and *w* variables, then the shaped surface be far from oval when the value of *w* is more and more large. Otherwise, if the value of *w* is more and more small, then the shaped surface close to ellipsoid. This conclusion can be satisfy if *a* and *b* are constant and sufficed the interval $0{,}681 \le \frac{b}{a} \le 0{,}815$.

If the values of *a* and *w* are constant and *b* variables, then the shaped surface close to sphere when the value of *b* close to *a*. Otherwise, when the value of *b* is more and more small, the shaped surface close to ellipsoid if the value of *w* close to zero, and the shaped surface close to oval if the value of *b* is in the interval of $0{,}681 \le \frac{b}{a} \le 0{,}815$. This conclusion is hold if the values of *a* and *w* are in the interval of $0{,}0292 \le \frac{w}{a} \le 0{,}168$.

If the values of *b* and *w* are constant and *a* variables, the shaped surface be far from oval when the value of *a* is more and more large. Meanwhile, if the values of $a = b > 0$ and $w = 0$, then the shaped surface is sphere. Whereas, if the values of $a > b > 0$ and $w = 0$, then the shaped surface is ellipsoid.

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