Kontribusi Fisika Indonesia
Vol. 13 No.1, Januari 2002

Perturbative Calculation for the Heavy Meson Matrix Element

L. T. Handoko
Kelompok Fisika Teoritik dan Komputasi, Fisika LIPI
Kompleks PUSPIPTEK Serpong, Tangerang 15310, Indonesia
E-mail: handoko@lipi.fisika.net

Abstract
We argue that the perturbative approach can be used to compute non-factorizable corrections to exclusive radiative B *meson decays in the heavy quark mass limit. The explicit computation of first order QCD corrections is given. These, or the so-called hard spectator, corrections is done up to $\mathcal{O}(\alpha_s)$ in the leading-twist approximation. These results are combined with the already done renormalization group effect in the appropriate Wilson coefficients, the vertex corrections and the annihilation contributions to obtain a complete $\mathcal{O}(\alpha_s)$ improvements to the lowest order decay width. The result is particularly applied to radiative decay* $B \to \rho\gamma$ *which the analysis is done for its branching ratio, isospin symmetry breaking and direct* CP *violation.*

Keywords : *perturbative, factorization, QCD, B meson, rare decay*

Abstrak
Kami mendiskusikan bahwa pendekatan perturbasi bisa dipakai untuk menghitung koreksi yang tidak bisa difaktorisasi terhadap peluruhan eksklusif meson B *pada aproksimasi berat kuark yang besar. Diberikan perhitungan untuk koreksi QCD orde pertama. Hal ini yang biasa disebut sebagai koreksi spektator keras dilakukan pada aproksimasi twist terendah sampai $\mathcal{O}(\alpha_s)$. Hasil ini dalam koefisien Wilson dikombinasikan dengan efek dari grup renormalisasi yang telah dilakukan sebelumnya, yaitu koreksi verteks dan anihilasi untuk mendapatkan lebar peluruhan yang komplet untuk orde terendah. Hasil ini khususnya diaplikasikan pada peluruhan radiatif* $B \to \rho\gamma$*, dimana analisis dilakukan terhadap branching ratio, isospin symmetry breaking dan CP violation.*

Kata kunci : *perturbasi, faktorisasi, QCD, meson B, peluruhan jarang*

The radiative *B* meson decays are very attractive because they occur first at loop level. It is believed that these decay modes are very sensitive to the new physics enter through the loop. On the other hand, the experiments have already succeded in measuring the radiative decays $B \to K^*\gamma$[1] and its inclusive mode $B \to X_s \gamma$[2]. It is, further, highly expected that the radiative decays $B \to \rho\gamma$ would also be measured as well. This decay is important to determine the inner angles in the Cabibbo-Kobayashi-Maskawa matrix, i.e. ρ and η in the Wolfenstein parameterization. In this paper, let us concentrate only on this particluar decay, even though the analysis is valid for other decay modes by replacing the quark flavor and its appropriate hadron final state as well.

The radiative decays can be measured inclusively over the hadronic final state, or exclusively by tagging a particular light hadron, typically a kaon. However, the inclusive measurement is experimentally much more difficult but theoretically simpler to interpret. In contrast, the easier detection of exclusive transition requires the development of a systematic theoretical framework.

Focusing on the decays $B \to \rho\gamma$, i.e. both its neutral and charged modes, is also motivated by non-trivial facts that in these decays the isospin violating ratio,

$$\Delta \equiv \frac{1}{2}\left(\Delta^{+0} + \Delta^{-0}\right), \tag{1}$$

where

$$\Delta^{\pm 0} \equiv \frac{\Gamma(B^\pm \to \rho^{\pm\gamma})}{2\Gamma(B^0 \to \rho^0\gamma)} - 1, \tag{2}$$

and the direct *CP* asymmetry[3],

$$\mathcal{A}_{CP} \equiv \frac{\Gamma(B^- \to \rho^-\gamma) - \Gamma(B^+ \to \rho^+\gamma)}{\Gamma(B^- \to \rho^-\gamma) + \Gamma(B^+ \to \rho^+\gamma)} \tag{3}$$

are non-zero[4].

The decays $B \to \rho\gamma$ are governed by the following amplitude

$$\mathcal{A} = \langle \rho(p_\rho) \,|\, \mathcal{H}_{\rm eff} \,|\, B(p_B) \rangle , \qquad (4)$$

where the effective Hamiltonian describes the radiative weak-transition $b \to d\gamma$

$$\mathcal{H}_{\rm eff} = \frac{G_F}{\sqrt{2}} \Big[\lambda_u^{(d)} (C_1(\mu)\mathcal{O}_1(\mu) + C_2(\mu)\mathcal{O}_2(\mu)) - \lambda_d^{(t)} C_7^{eff}(\mu)\mathcal{O}_7(\mu) + \cdots \Big]. \qquad (5)$$

Here, $\lambda_q^{(q')} = V_{qb}V_{qq'}^*$ are the CKM factors, and we have restricted ourselves to those contributions which will be important in what follows. The operators $\mathcal{O}_1(\mu)$ and $\mathcal{O}_2(\mu)$ are the four-quark operators

$$\mathcal{O}_1 = (\bar{d}_\alpha \Gamma^\mu u_\beta)(\bar{u}_\beta \Gamma_\mu b_\alpha) , \qquad (6)$$

$$\mathcal{O}_2 = (\bar{d}_\alpha \Gamma^\mu u_\alpha)(\bar{u}_\beta \Gamma_\mu b_\beta) , \qquad (7)$$

$$\mathcal{O}_7 = \frac{e m_b}{8\pi^2} \, \bar{d} \sigma^{\mu\nu} (1-\gamma_5) \, F_{\mu\nu} b, \qquad (8)$$

where $\Gamma_\mu = \gamma_\mu (1-\gamma_5)$, α and β are the SU(3) color indices, C_1 and C_2 are the corresponding Wilson coefficients, and $F_{\mu\nu}$ is the electromagnetic field strength tensor.

The renormalization group effects and the vertex corrections enter in the Wilson coefficient C_7[5], while the annihilation contributions (including the long distance effects) appear as the linear combination of C_1 and C_2[6]. The hard spectator amplitude (HSA) turns out from the gluon emission in either b or d quark line connected to the spectator light quark line. It can be calculated in the form of a convolution formula whose leading $O(\alpha_s)$ term can be expressed as[7],

$$M^{HSA} = \frac{4\pi\alpha_s C_F}{N_c} \int_0^1 du \int_0^1 d\ell_+ M_{jk}^B M_{li}^\rho T_{ijkl}, \qquad (9)$$

where N_c is the number of colours,

$$C_F = \frac{(N_c^2 - 1)}{(2N_c)}, \qquad (10)$$

is the Casimir operator eigenvalue in the fundamental representation of $SU(N_c)$ group, and T_{ijkl} is the hard scattering amplitude. According to this convolution formula, it is argued that this so-called non-factorizable contributions can be calculated perturbatively, because the hard scattering amplitude is calculable order by order. This technique has been done and proved up to $O(\alpha_s^2)$ accuracy originally for the hadronic B decays[8]. However this proof is valid in the leading order of the inverse of the B meson mass, and the leading twist approximation for the two particle light-cone projector operators, M_{jk}^{B} and M_{li}^{ρ}.

The hard spectator corrections contribute to the branching ratio significantly, i.e. enhance the magnitude ~10% in the whole region[9]. This enhancement is reduced very much in the isospin symmetry breaking, since the quantity is by definition normalized by the same effect as can be seen in Eq. (1). On contrary, the contribution is meaningless in the *CP* asymmetry since the hard spectator corrections up to $O(\alpha_s^2)$ do not induce any additional strong phase. At $O(\alpha_s^2)$ accuracy, however, there would be slightly changed. This will be discussed in the subsequent work.

References

1. R. Ammar et.al. (CLEO Collaboration), *Phys. Rev. Lett.* **71** (1993) 674; T.E. Coan et.al. (CLEO Collaboration), *Phys. Rev. Lett.* **84** (2000) 5283; B. Aubert et.al. (BABAR Collaboration), BABAR-CONF-00-12 (2000).
2. M.S. Alam et al. (CLEO Collaboration), *Phys. Rev. Lett.* **74** (1995) 2885; S. Ahmed et al. (CLEO Collaboration), CLEO-CONF 99-10 (1999); R. Barate et.al. (ALEPH Collaboration), *Phys. Lett.* **B429** (1998) 169; M. Nakako et.al. (Belle Collaboration), BELLE-CONF-003 (2000).
3. For example see : L. T. Handoko and J. Hashida, *Phys. Rev*. **D58** (1998) 094008.
4. Ali, L.T. Handoko and D. London, *Phys. Rev*. **D63** (2000) 014014; L.T. Handoko, *Nucl. Phys. Suppl*. **B93** (2001) 296.
5. Ali and C. Greub, *Z. Phys*. **C49** (1991) 431; K. Adel and Y-P. Yao, *Phys. Rev*. **D49** (1994) 4945; C. Greub, T. Hurth, and D. Wyler, *Phys. Rev*. **D54** (1996) 3350; K. Chetyrkin, M. Misiak, M. Münz, *Phys. Lett*. **B400** (1997) 206; Err. ibid. B425 (1998) 414.
6. Ali and V.M. Braun, *Phys. Lett*. **B359** (1995) 223; A. Khodjamirian, R. Ruckl, G. Stoll and D. Wyler, *Phys. Lett*. **B402** (1997) 167; B. Grinstein and D. Pirjol, hep-h/0002216 (2000).
7. M. Beneke, Th. Feldmann and D. Seidel, *hep-ph/0106067* (2000).
8. M. Beneke, G. Buchalla, M. Neubert and C.T. Sachrajda, *Phys. Rev. Lett.* **83** (1999) 1914;
9. Nucl. Phys. **B591** (2000) 313; M. Beneke and Th. Feldmann, *Nucl. Phys*. **B592** (2001) 3.
10. Ali and A. Ya Parkhomenko, *hep-ph/0105302* (2001).