Indonesian Journal of Physics
Vol. 26. No.1, July, 2015
Journal homepage: http://ijphysics.com

Filter Design on Discrete-Time Neutral System Using Guaranteed Cost Method

Erwin Susanto[1], Ig. Prasetya Dwi Wibawa[3] and Junartho Halomoan[2],
[1,2,3]School of Electrical Engineering, Telkom University

Received: 27 May 2015, Revised: 29 July 2015, Accepted: 28 July 2015

Abstract

This paper will discuss about our study in filter design on discrete-time neutral system using guaranteed cost method. This design filter method yields a robust filter and the derivation of its equation can be obtained by using LMI (linear matrix inequalities).

Keywords: guaranteed cost, LMI, robust control

INTRODUCTION

Uncertainty in a system shows impact on stability and performance of control system and signal processing. Therefore, how to design a robust filter that will guarantee adequate level of performance becomes main problem in many applications. Common problems that often met on designing a filter are disturbance, perturbation on the output because of uncertainty on the system [1]. Guaranteed cost method will be proposed to solve these problems [2-3].

LMI approachment had been used and had a big contribution in term of solving complex issues on control system related to get proper solution for convergence optimation [4].

Design of a system based on control theory and its application can be categorized into two systems, i.e. neutral system and retarded system. Neutral system, which its dynamics depends on delay condition and its derivatives. This system can be found in some process control and dynamics process system [5-7].

Some researches in the field of filter design are also considered of robust system analysis [1], [8-10]. Filter design system on previous researches are mostly continous system. However, discrete/digital system can be implemented easier on a real condition. Therefore, on this paper we will discuss on design and analysis of discrete time robust filter using guaranteed cost method based on LMI approach [11]. System stability can be explained by using Lyapunov equation.

Notations that are used on this paper as follow: uppercase letter of "T" and superscript "-1" imply transpose and inverse matrix. $\Re^n$ denotes Eucledian's distance of n-dimensional space. Meanwhile, $X > Y$ or $X \geq Y$ states $X-Y$ is always positive definite or positive semidefinite. I is the appropriate identity matrix and * denotes symmetric element of symmetrical matrix.

SYSTEM DESIGN

A discrete-time neutral system with uncertainty can be modeled as follow

$$x(k+1) = (G + \Delta G(k))x(k) + (G_d + \Delta G_d(k))x(k-h) + (G_n + \Delta G_n(k))x(k+1-\tau) \quad (1)$$

$$y(k) = Cx(k) \quad (2)$$

$$x(k) = \psi(k), k \in \left[-\max\{h,\tau\},0\right] \quad (3)$$

where h is time delay, τ is neutral delay, $x(k) \in \Re^n$ is state vector, $y(k) \in \Re^n$ is measured output vector, $G, G_d, G_n \in \Re^n$ is suitable real

* Corresponding author.
E-mail address: irawan_dani@yahoo.com

constant matrix, where denotes neutral time delay system, $x(k) = \psi(k)$ denotes continuous vector of initial function, and C is real constant matrix with certain value with appropriate dimension.

Matrices $\Delta G(k), \Delta G_d(k), \Delta G_n(k)$ are norm-bounded real matrix functions that denote uncertainty parameter. Let

$$\Delta G(k) = D_G F_G(k) E_G, \Delta G_d(k) = D_{Gd} F_{Gd}(k) E_{Gd}, \quad \Delta G_n(k) = D_{Gn} F_{Gn}(k) E_{Gn} \tag{4}$$

where $F_G(k), F_{Gd}(k), F_{Gn}(k)$ are uncertain matrix that depend on time and suite these inequalities as follow

$$F_G^T(k) F_G(k) \le I, F_{Gd}^T(k) F_{Gd}(k) \le I,$$

$$F_{Gn}^T(k) F_{Gn}(k) \le I$$

$D_G, D_{Gd}, D_{Gn}, E_G, E_{Gd}, E_{Gn}$ are real constant matrices with certain value that has appropriate dimension.

We design filter that asymptotically stable on the equations [1]-[3] as follow

$$\hat{x}(k+1) = M\hat{x}(k) + Ly(k) \tag{5}$$

where M and L are state variable of the filter.

Let error state vector and its derivation are modeled as follow

$$e(k) = x(k) - \hat{x}(k) \tag{6}$$

$$e(k+1) = x(k+1) - \hat{x}(k+1)$$

$$\begin{aligned} e(k+1) = Me(k) &+ ((G + \Delta G(k)) - LC - M)x(k) \\ &+ (G_d + \Delta G_d(k))x(k-h) \\ &+ (G_n + \Delta G_n(k))x(k+1-\tau) \end{aligned} \tag{7}$$

Furthermore, define augmented matrix state vector as follow

$$x_a(k) = \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} \tag{8}$$

so that

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} (G+\Delta G) & 0 \\ (G+\Delta G - LC - M) & M \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} (G_d + \Delta G_d) & 0 \\ (G_d + \Delta G_d) & 0 \end{bmatrix} \begin{bmatrix} x(k-h) \\ e(k-h) \end{bmatrix} + \begin{bmatrix} (G_n + \Delta G_n) & 0 \\ (G_n + \Delta G_n) & 0 \end{bmatrix} \begin{bmatrix} x(k+1-\tau) \\ e(k+1-\tau) \end{bmatrix}$$

$$\begin{bmatrix} x(k+1) \\ e(k+1) \end{bmatrix} = \begin{bmatrix} (G+\Delta G) & 0 \\ (G+\Delta G - LC - M) & M \end{bmatrix} \begin{bmatrix} x(k) \\ e(k) \end{bmatrix} + \begin{bmatrix} (G_d + \Delta G_d) & 0 \\ (G_d + \Delta G_d) & 0 \end{bmatrix} \begin{bmatrix} x(k-h) \\ e(k-h) \end{bmatrix} + \begin{bmatrix} (G_n + \Delta G_n) & 0 \\ (G_n + \Delta G_n) & 0 \end{bmatrix} \begin{bmatrix} x(k+1-\tau) \\ e(k+1-\tau) \end{bmatrix} \tag{9}$$

Estimation of signal and upper boundary of guaranteed cost function that are minimized as follow

$$z(k) = Ke(k) \tag{10}$$

$$J = \sum_{k=-\infty}^{\infty} z^T(k) z(k) \tag{11}$$

where $z(k)$ is output state error and K is matrix with its weighting factor is determined before. So that

$$x_a(k+1) = G_1 x_1(k) + G_2 x_2(k-h) + G_3 x_3(k+1-\tau) \tag{12}$$

$$z(k) = C_1 x_a(k) \; z(k) = C_1 x_a(k) \tag{13}$$

where

$$G_1(k) = \begin{bmatrix} (G+\Delta G) & 0 \\ (G + \Delta G - LC - M) & G \end{bmatrix},$$

$$G_2(k) = \begin{bmatrix} (G_d + \Delta G_d) & 0 \\ (G_d + \Delta G_d) & 0 \end{bmatrix},$$

$$G_3(k) = \begin{bmatrix} (G_n + \Delta G_n) & 0 \\ (G_n + \Delta G_n) & 0 \end{bmatrix}, \quad C_1 = \begin{bmatrix} 0 & K \end{bmatrix}$$

On the next equation, we use lemmas to prove the equations.

Lemma 1. (see [5]). Let D and E are matrices with appropriate dimension, and F is matrix function that satisfies the equation $F^T(k) F(k) \le I$ so for certain scalar value α, will satisfy inequality as follow

$$DFE + E^T F^T D^T \le \alpha DD^T + \alpha^{-1} E^T E \tag{14}$$

RESULTS AND DISCUSSION

On this part, sufficient conditions are required related to the existence of discrete time guaranteed cost filter. The main result of this study can be obtained by using Theorem 1.

Theorem 1.

Optimation problem and its constraints are given by these inequalities

$$\min\{\gamma_1 + \gamma_2 + tr(J_1 + J_2 + J_3 + J_4)\}$$

and constraints

$$\begin{bmatrix} -E + W + C_1^T C_1 & 0 & 0 & G_1^T \\ * & -W & 0 & G_2^T \\ * & * & -S & G_3^T \\ * & * & * & -(E+S)^{-1} \end{bmatrix} < 0 \tag{15}$$

$$\begin{bmatrix} -\gamma_1 & x^T(0) \\ * & -E_{x(inv)} \end{bmatrix} < 0, \quad \begin{bmatrix} -\gamma_2 & e^T(0) \\ * & -E_{e(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -J_1 & Y_1^T(0) \\ * & -W_{x(inv)} \end{bmatrix} < 0, \begin{bmatrix} -J_2 & Y_2^T(0) \\ * & -W_{e(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -\gamma_1 & x^T(0) \\ * & -E_{x(inv)} \end{bmatrix} < 0, \begin{bmatrix} -\gamma_2 & e^T(0) \\ * & -E_{e(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -M_1 & Y_1^T \\ * & -W_{x(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -J_3 & Y_3^T(0) \\ * & -S_{x(inv)} \end{bmatrix} < 0, \begin{bmatrix} -J_4 & Y_4^T(0) \\ * & -S_{e(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -M_2 & Y_2^T \\ * & -W_{e(inv)} \end{bmatrix} < 0, \begin{bmatrix} -M_3 & Y_3^T \\ * & -S_{x(inv)} \end{bmatrix} < 0,$$

$$\begin{bmatrix} -M_4 & Y_4^T \\ * & -S_{e(inv)} \end{bmatrix} < 0, \quad (16)$$

with set of solutions as follow

$$E>0, S>0, W>0, E_x>0, E_e>0, W_x>0, W_e>0,$$
$$S_x>0, S_e>0, J_1>0, J_2>0, J_3>0, J_4>0,$$
$$\gamma_1, \gamma_2, tr(J_1), tr(J_2), tr(J_3), tr(J_4). \quad (16)$$

where

$$E_{inv} = E^{-1}, S_{inv} = S^{-1}, E_{inv} = E^{-1} W_{inv} = W^{-1}.$$

so that the system on equation (5) is guaranteed cost filter with upper boundary as follow

$$J = x^T(0)E_x x(0) + e^T(0)W_e e(0)$$
$$+ \sum_{i=-h}^{-1} x^T(i)W_x x(i) + \sum_{i=-h}^{-1} e^T(i)W_e e(i)$$
$$+ \sum_{s=-\tau}^{0} x^T(s)S_x x(s) + \sum_{s=-\tau}^{0} e^T(s)S_e e(s)$$
$$= \gamma_1 + \gamma_2 + tr(J_1) + tr(J_2) + tr(J_3) + tr(J_4) \quad (17)$$

Note 1: LMI approach can be obtained by using iteration in order to satisfy inverse matrix corresponds with equation (15) and equation (16) [1].

Proof of Theorem 1.

Define the candidate of Lyapunov function as follow

$$V(x_a(k)) = x_a^T(0)Ex_a(0) + \sum_{i=k-h}^{k-1} x_a^T(i)Wx_a(i)$$
$$+ \sum_{s=k-\tau}^{k} x_a^T(s)Sx_a(s) \quad (18)$$

where $E = \begin{bmatrix} E_x & 0 \\ * & E_e \end{bmatrix}, W = \begin{bmatrix} W_x & 0 \\ * & W_e \end{bmatrix}$, and $S = \begin{bmatrix} S_x & 0 \\ * & S_e \end{bmatrix}$.

Difference of equation (18) according to equation (1) can be formulated as below

$$\Delta V = V(x_a(k+1)) - V(x_a(k))$$
$$= x_a^T(k+1)Ex_a(k+1) - x_a^T(k)Ex_a(k)$$
$$+ x_a^T(k)Wx_a(k) - x_a^T(k-h)Wx_a(k-h)$$
$$+ x_a^T(k+1)Sx_a(k+1) - x_a^T(k-\tau)Sx_a(k-\tau) \quad (19)$$

For providing asymptotic stability and for minimizing guaranteed cost on dynamics error process, we use the Lyapunov's inequality below

$$\Delta V < -z^T(k)z(k) < 0 \quad (20)$$

Let

$$\psi = \begin{bmatrix} \psi_1 & \psi_2 & \psi_3 \\ * & \psi_4 & \psi_5 \\ * & * & \psi_6 \end{bmatrix}, \text{ hence}$$

$$\begin{bmatrix} x_a(k) \\ x_a(k-h) \\ x_a(k+1-\tau) \end{bmatrix}^T \begin{bmatrix} \psi_1 & \psi_2 & \psi_3 \\ * & \psi_4 & \psi_5 \\ * & * & \psi_6 \end{bmatrix} \begin{bmatrix} x_a(k) \\ x_a(k-h) \\ x_a(k+1-\tau) \end{bmatrix} < 0 \quad (21)$$

where

$$\psi_1 = G_1^T(E+S)G_1 - E + W + C_1^T C_1,$$
$$\psi_2 = G_1^T(E+S)G_2, \quad \psi_3 = G_1^T(E+S)G_3,$$
$$\psi_4 = G_2^T(E+S)G_2, \quad \psi_5 = G_2^T(E+S)G_3$$
$$\psi_6 = G_2^T(E+S)G_3 - S$$

For condition $\psi < 0$, by using Schur Complement [4], we can derive the equation as follow

$$\begin{bmatrix} -E+W+C_1^T C_1 & 0 & 0 & G_1^T(E+S) \\ * & -W & 0 & G_2^T(E+S) \\ * & * & -S & G_3^T(E+S) \\ * & * & * & -(E+S) \end{bmatrix} < 0 \quad (22)$$

and by using multiplication matrix, before and after, with $diag\{I, I, I, (E+S)^{-1}\}$, we have

$$\begin{bmatrix} -E+W+C_1^TC_1 & 0 & 0 & G_1^T \\ * & -W & 0 & G_2^T \\ * & * & -S & G_3^T \\ * & * & * & -(E+S)^{-1} \end{bmatrix} < 0 \quad (23)$$

Furthermore, equation (20) will be used as substitute on the following inequality

$$V(k+1) < \sum_{k=0}^{\infty} -z^T(k)z(k) < 0 \quad (24)$$

give result as follow

$$J = \sum_{k=0}^{\infty} z^T(k)z(k) < x^T(0)Ex(0) + e^T(0)W_e e(0)$$
$$+ \sum_{i=-h}^{-1} x^T(i)W_x x(i) + \sum_{i=-h}^{-1} e^T(i)W_e e(i)$$
$$+ \sum_{s=-\tau}^{0} x^T(s)S_x x(s) + \sum_{s=-\tau}^{0} e^T(s)S_e e(s) = J^* \quad (25)$$

where J^* is guaranteed cost, and each element from the equation (25) is defined as follow

$$\sum_{i=-h}^{-1} x(i)x^T(i) = Y_1Y_1^T,$$
$$\sum_{i=-h}^{-1} e(i)e^T(i) = Y_2Y_2^T,$$
$$\sum_{s=-\tau}^{0} x(s)x^T(s) = Y_3Y_3^T,$$
$$\sum_{s=-\tau}^{0} e(s)e^T(s) = Y_4Y_4^T. \quad (26)$$

Obviously that the equation (26) refers on the equation (16). Hence, the robust filter design suites the model.

For numerical example and its implementation will be conducted on the following study.

REFERENCES

[1] E. Susanto and M. Ishitobi, "An LMI approach to guaranteed cost filter for uncertain neutral systems with time-varying delay", Proc. of 2013 International Conference on Advanced Computer Science and Information Systems, Bali, 2013, pp. 125-130.

[2] S.S.L. Chang, and T.K.C Peng, "Adaptive guaranteed cost control of systems with uncertain parameters," IEEE Transactions on Automatic Control, Vol. 7, 1972, pp. 474–483.

[3] J.H. Kim, "Robust guaranteed cost filtering for uncertain systems with time-varying delay via LMI approach," Transactions on Control, Automation and Systems Engineering, Vol. 3.No 1, 2001, pp. 27–31.

[4] S. Boyd, El. Ghaoui, E. Feron, and V. Balakrishnan, Linear *Matrix Inequalities in System and Control Theory*, SIAM, Philadelphia, 1994.

[5] C.H. Lien, "Guaranteed cost observer-based controls for a class of uncertain neutral time-delay systems," Journal of Optimization Theory and Applications, Vol. 126, 2005, pp. 137–156.

[6] S. Xu, J. Lam, C. Yang, and E. Verriest, "An LMI approach to guaranteed cost control for uncertain linear neutral delay systems," International Journal of Robust and Nonlinear Control, Vol. 13, 2003, pp. 35–53.

[7] K-W. Yu, and C.H. Lien, "Delay-dependent conditions for guaranteed cost observer-based control of uncertain neutral systems with time varying delays," IMA Journal of Mathematical Control and Information, Vol. 24, 2007, pp. 383–394.

[8] Y. He, G.P. Liu, D. Rees, and M. Wu, "Improved H∞ filtering for systems with a time-varying delay," Circuits System Signal Process, Vol. 29, 2010, pp. 377–389.

[9] J. Yoneyama, "Robust H∞ filter design for uncertain systems with time-varying delays," Electronic and Communication in Japan, Vol. 128-C, 2010, pp. 970–975.

[10] E. Fridman, and U. Shaked, "An improved delay-dependent H∞ filtering of linear neutral systems," IEEE Transactions on Signal Processing, Vol. 52, 2004, pp. 668–673.

[11] J.H. Kim, "Discrete-time robust guaranteed cost filtering for convex bounded uncertain systems with time delay," ICASE: The Institute of Control, Automation and Systems Engineering, Korea Vol. 4. No.4, 2002, pp. 324–329.