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Effect of a Transverse Magnetic Field on Buoyancy-Driven Flow and Heat Transfer in a Porous Trapezoidal Enclosure

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Abstract

The effect of a magnetic field on buoyancy-driven flow and heat transfer in a trapezoidal enclosure filled with a fluid-saturated porous medium is studied numerically using the finite difference method. The inclined sloping boundaries are treated by adopting staircase-like zigzag lines. The sloping walls are maintained isothermally at different temperatures. The top and bottom horizontal straight walls are kept adiabatic. The results indicate that the heat is transferred almost entirely by pure conduction with a sufficiently large magnetic field. Utilitizing the square geometry is more effective to suppress the heat transfer rate than the trapezoidal geometry.

Keywords: *Natural convection; Porous media; Darcy's law; Trapezoidal enclosure.*

1. Introduction

Convective flows in porous media have occupied the central stage in many fundamental heat transfer analysis and has received considerable attention over the last few decades. This interest is due to its wide range of applications, for example, high performance insulation for buildings, chemical catalytic reactors, packed sphere beds, grain storage and such geophysical problems as frost heave. Porous media are also of interest in relation to the underground spread of pollutants, solar power collectors, and to geothermal energy systems.

Most of the published papers are concerned with the analysis of buoyancy-driven flow and heat transfer in square/rectangular enclosures filled with porous media; see, for example, Manole and Lage[1)], Goyeau *et al*.[2)] and Saeid and Pop[3)]. In reality, natural convection in a differentially heated enclosure is a prototype of many industrial applications and in particular, a trapezoidal enclosure has received considerable attention because of its applicability in various fields. The moderately concentrating solar energy collector is an important example involving a trapezoidal geometry. The absorbing surface and side walls are enclosed by the addition of cover plate. The cover plate, which is at the lower temperatures than the absorber is used to suppress convection and radiation heat losses. The side walls are reflective surfaces and could contribute the overall convective field in the groove.

The study of convective flow in a trapezoidal geometry is more difficult than that of square or rectangular enclosures due to the presence of sloping walls. In general, the mesh nodes do not lie along the sloping walls and consequently, from a programming and computational point of view, the effort required for determining flow characteristic increases significantly. Relatively little work has been done in a porous trapezoidal geometry. Kumar and Kumar[4)] applied finite element method with GMRES, a Krylov subspace based solver to solve natural convection in a porous trapezoidal enclosure. Varol and Oztop[5)] solved the problem by finite difference method, regular rectangular grid and adopting staircase-like zigzag lines for the inclined boundaries. However, they[4,5)] did not considered effect of a magnetic field.

Bian *et al*.[6)] investigated the effect of a transverse magnetic field on natural convection in an inclined porous rectangular enclosure. In particular, they found that the magnetic field has a profound effect on the transition angle from a single-cell to a multiple-convection pattern. This study was later extended by Grosan et al.[7)] to study an effect of an inclined magnetic field when internal heat generation takes into account. It was shown that both the strength and inclination angle of the magnetic field have a strong influence on convection modes. To the best of our knowledge, investigation of the effects of a magnetic field on natural convection in a trapezoidal enclosure filled with a fluid-saturated porous medium has not been undertaken yet. The purpose of the present paper is therefore, to investigate the effects of a magnetic field on buoyancy-driven flow or natural convection in a porous trapezoidal enclosure.

2. Mathematical formulation

We consider the steady, two-dimensional natural convection flow in a trapezoid region filled with an electrically conducting fluid-saturated porous medium, see Figure 1(a). The co-ordinate system employed is also depicted in this figure. The top and bottom surfaces of the convective region are assumed to be thermally insulated and the sloping surfaces to be heated and cooled at constant temperatures T_h and T_c, respectively. θ_s is the inclination angle of the sloping walls. θ_s = 90° means that enclosure is a square with width L.

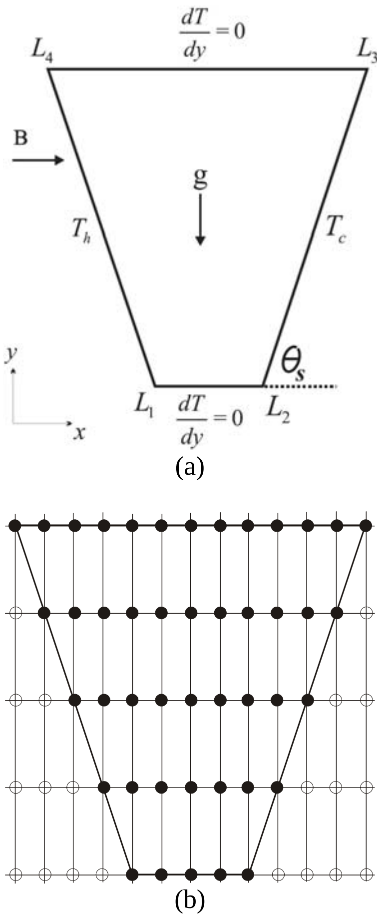

Figure 1. Schematic representation of the model (a), Grid-points distribution (b).

A uniform and constant magnetic field $\vec{B}$ is applied normal to gravity direction. The viscous, radiation and Joule heating effects are neglected. The resulting convective flow is governed by the combined mechanism of the driven buoyancy force and the retarding effect of the magnetic field. The magnetic Reynolds number is assumed to be small so that the induced magnetic field can be neglected in favor of the applied magnetic field.

Under the above assumptions, the conservation equations for mass, momentum under the Darcy approximation, energy and electric transfer are given by:

$$\nabla\cdot\vec{V}=0, \tag{1}$$

$$\vec{V}=\frac{K}{\mu}(-\nabla P+\rho\vec{g}+\vec{I}\times\vec{B}), \tag{2}$$

$$(\vec{V}\cdot\nabla)T=\alpha_m\nabla^2 T, \tag{3}$$

$$\nabla\cdot\vec{I}=0, \tag{4}$$

$$\vec{I}=\sigma(-\nabla\varphi+\vec{V}\times\vec{B}), \tag{5}$$

$$\rho=\rho_0[1-\beta(T-T_c)] \tag{6}$$

where $\vec{V}=(u,v)$ is the fluid velocity vector, T is the fluid temperature, P is the pressure, $\vec{B}$ is the external magnetic field, $\vec{I}$ is the electric current, φ is the electric potential, $\vec{g}$ gravitational acceleration vector, K is the permeability of the porous medium, α_m is the effective thermal diffusivity, ρ is the density, μ is the dynamic viscosity, β is the coefficient of thermal expansion, c_p is the specific heat at constant pressure, σ is the electrical conductivity, ρ_0 is the reference density and $-\nabla\varphi$ is the associated electric field. As discussed by Garandet *et al.*[8], Eqs. (4) and (5) reduce to $\nabla^2\varphi=0$. The unique solution is $-\nabla\varphi=0$ since there is always an electrically insulating boundary around the enclosure. Thus, it follows that the electric field vanishes everywhere[9]. Furthermore, eliminating the pressure term in Eq. (2) in the usual way then the governing Eqs. (1) - (6) can be written as

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0, \tag{7}$$

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{gK\beta}{\nu}\frac{\partial T}{\partial x}-\frac{\sigma KB_0^2}{\mu}\left(\frac{\partial u}{\partial y}\right), \tag{8}$$

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_m\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right). \tag{9}$$

where B_0 is the magnitude of $\vec{B}$ and ν is the kinematic viscosity of the fluid. The following are the boundary conditions:

$$\text{on } \ell_2\ell_3:\quad u=0,\quad T=T_c,$$

$$\text{on } \ell_1\ell_4:\quad u=0,\quad T=T_h,$$

$$\text{on } \ell_1\ell_2,\ell_3\ell_4:\quad v=0,\quad \frac{\partial T}{\partial y}=0. \tag{10}$$

Now we introduce the following non-dimensional variables

$$X=\frac{x}{\ell},Y=\frac{y}{\ell},U=\frac{\ell}{\alpha_m}u,V=\frac{\ell}{\alpha_m}v,\Theta=\frac{T-T_c}{\Delta T} \tag{11}$$

Introducing the stream function Ψ defined as $U=\partial\Psi/\partial Y$ and $U=-\partial\Psi/\partial X$, and using expressions (11) in Eqs. (7) - (9), we obtain the following partial differential equations in non-dimensional form:

$$(1+Ha^2)\frac{\partial^2\Psi}{\partial X^2}+\frac{\partial^2\Psi}{\partial Y^2}=-Ra\frac{\partial\Theta}{\partial X}, \tag{12}$$

$$\frac{\partial^2\Theta}{\partial X^2}+\frac{\partial^2\Theta}{\partial Y^2}=\frac{\partial\Psi}{\partial Y}\frac{\partial\Theta}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\Theta}{\partial Y}. \tag{13}$$

Subject to the boundary conditions

$$\text{on } L_2L_3:\ \Psi=0,\ \Theta=0,$$

$$\text{on } L_1L_4:\ \Psi=0,\ \Theta=1,$$

$$\text{on } L_1L_2, L_3L_4:\ \Psi=0,\ \frac{\partial\Theta}{\partial Y}=0. \tag{14}$$

where $Ra=gK\beta L\Delta T/(\alpha_m\nu)$ is the Rayleigh number, and $Ha=\sigma KB_0^2/\mu$ is the Hartman number for the porous medium. Once we know the temperature we can obtain the rate of heat transfer from the hot sloping wall, which is given in terms of the mean Nusselt number at the hot wall as

$$Nu_m = -\int_{L_1}^{L_1} \frac{\partial \Theta}{\partial X}\, \mathrm{d}Y \quad (15)$$

3. Finite Difference Method

We employed the finite difference method to solve Eqs. (12) and (13) subject to (14). The central difference method was applied for discreetizing the equations. The solution of the algebraic equations was performed using the Gauss-Seidel iteration with relaxation method. The unknowns Ψ and Θ are calculated until the following convergence criterion is fulfilled:

$$\max\left[\frac{\left|\zeta_{i,j}^{n+1} - \zeta_{i,j}^{n}\right|}{\left|\zeta_{i,j}^{n+1}\right|}\right] \le \varepsilon, \quad (15)$$

where ζ is either Ψ or Θ, n represents the iteration number and ε is the convergence criterion. In this study, the convergence criterion is set at $\varepsilon = 10^{-6}$. The mean Nusselt number in Eq. (15) was calculated using the integration trapezoidal rule.

A regular rectangular grid is placed over the trapezoidal enclosure. Selecting an appropriate aspect ratio and density is compulsory to maintain the sloping boundaries match exactly at the nodal point as displayed in Figure 1 (b). The solution domain of the trapezoidal which equations are applied, denoted by bold node. The typical numerical run used was a 33 x 100 (θ_s = 72^o) grid. We also performed a few runs with 65 x 197 and 129 x 391 grids to check the accuracy, finding good agreement with runs using the 33x100 for the same parameter values. As a validation, our results for the mean Nusselt number for the case θ_s = 72^o and untitled trapezoidal enclosure compares well with that obtained by Varol and Oztop[5], in the absence of a magnetic field (Table 1). Table 1 also shows the good agreement between our result and the existing results for a porous square (θ_s = 90^o) enclosure without magnetic field effect.

Table 1. Comparison of the Nu_m for some results from the literature at Ra = 1000.

θ_s	References	Nu_m
	Varol and Oztop[5]	9.170
72^o	Present result	9.158
	Manole and Lage[1]	13.637
	Goyeau et al.[2]	13.470
	Saeid and Pop[3]	13.726
	Varol and Oztop[5]	13.564
90^o	Present result	13.199

4. Result and Discussion

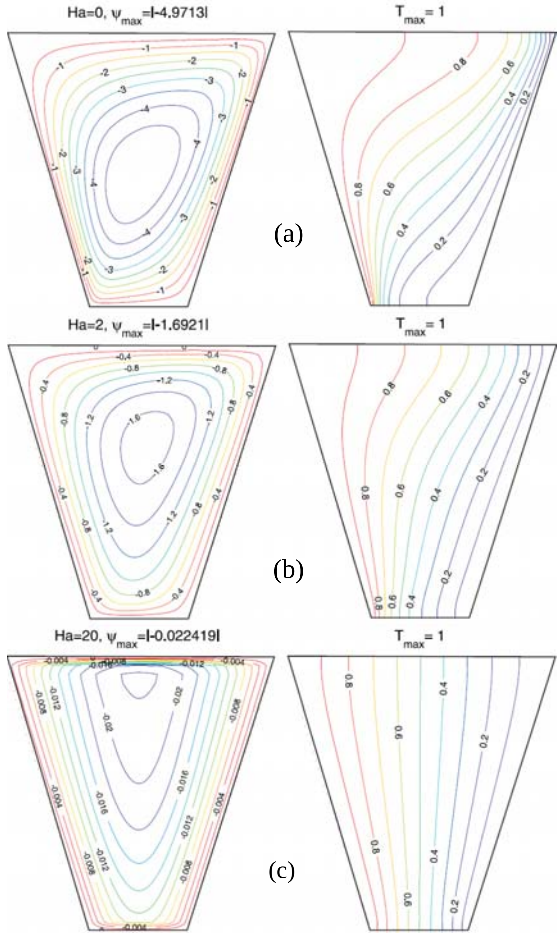

Figure 2. Contour plots of the stream function and temperature for Ra = 100, θ_s = 72^o and Ha = 0 (a), Ha = 2 (b), Ha = 20 (c).

Figure 2 shows the evolutions of the fuid motion and the distribution of heat for $Ra = 100$ and different magnetic fields with an inclination angle of the sloping wall, θ_s = 72^o. The fluid motion as shown in the figure is described as follows. Since the temperature of the left wall is higher than that of the fluid inside the enclosure, the wall transmits heat to the fluid and raises the temperature of fluid particles adjoining the left wall. When the temperature rises, the fluid starts moving from the left wall (hot) to the right wall (cold) and falling along the cold wall, then rising again at the hot wall, creating a clockwise rotating cell inside the enclosure. When the magnetic field is imposed on the enclosure, the intensity of convective motion weakens significantly (known from absolute value of Ψ_{max}). The core of the vortex is towards the top wall; see Figure 2(b), 2(c). The isotherms are almost perpendicular with the horizontal wall as Ha increase. This indicates that heat is transferred almost entirely by pure conduction with a sufficiently large magnetic field.

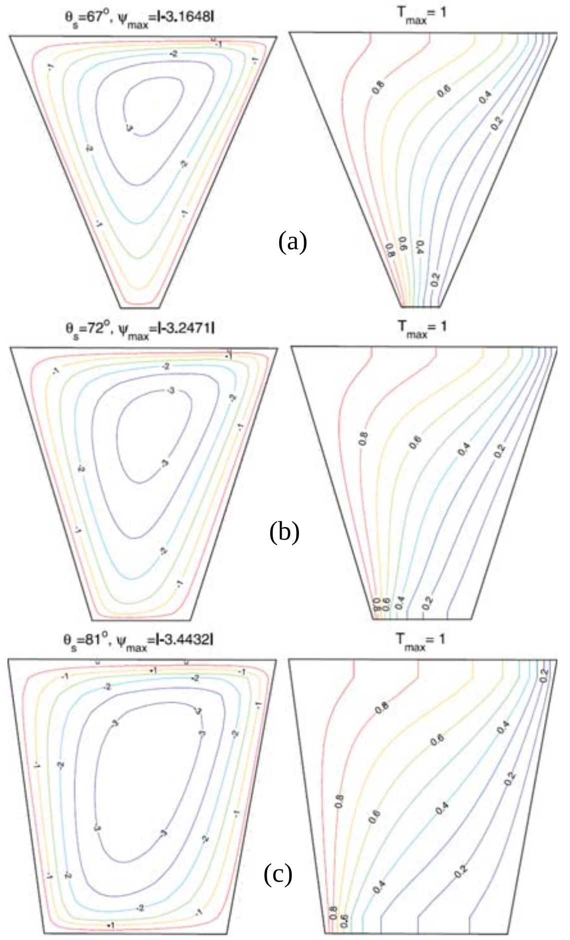

Figure 3. Contour plots of the stream function and temperature for Ha = 5, Ra = 100 and θ_s = 67^o (a), θ_s = 72^o (b), and θ_s = 81^o (c).

The effects of θ_s on the stream function and isotherm are shown in Figure 3 for Ha = 5 and Ra = 1000. This figure reveals that θ_s is not having the major effect on the flow and temperature pattern, where only a little increasing in convective motion occurred.

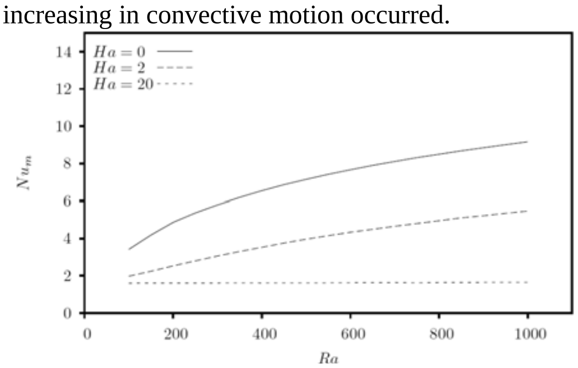

Figure 4. Plots of the Nu_m against Ra. for the values of Ha. labelled on the figure with θ_s = 72^o.

Figure 4 demonstrates the relationship between the mean Nusselt number and the Rayleigh number for the case θ_s = 72^o and different values of Ha. Naturally, the heat transfer increases with increasing Ra. The rate of heat transfer is progressively reduced by the presence of a magnetic field. Furthermore, for a sufficiently large magnetic field, increasing Ra no longer effect to the heat transfer rate.

The variations of the mean Nusselt number with Ha. for different values of θ_s are shown in Figure 5. In general, the Nu_m initially decreases steeply with Ha. As the value of Ha is made larger, the strength of the heat transfer is progressively suppressed and the Nu_m goes to fixed value. We observed that θ_s = 67^o is the most effective to suppress the heat transfer rate when magnetic field is neglected. However for a sufficient large magnetic, θ_s = 81^o is the most effective to suppress the heat transfer rate.

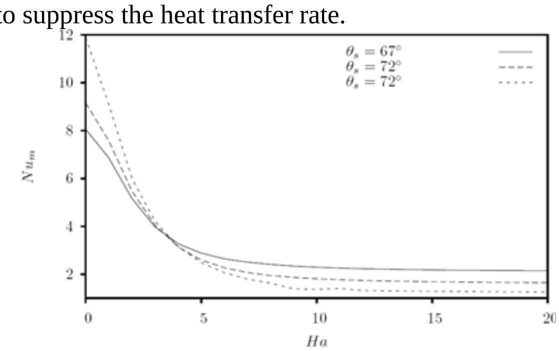

Figure 5. Plots of the Nu_m against Ha. for the values of θ_s labelled on the figure with Ra = 1000.

4. Conclusion

The present numerical study exhibits several interesting features concerning the effect of the transverse magnetic fields on buoyancy-driven flow and heat transfer in a trapezoidal enclosure filled with a porous medium. The main conclusions of the present analysis are as follows. The heat is transferred almost entirely by pure conduction with a sufficiently large magnetic field. Utilitizing square enclosure is more effective to suppress the heat transfer rate than the trapezoidal enclosure. A discussion on a more general configuration will constitute the subject of our subsequent investigation to treat more complex problems, such as time-dependent flows.

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