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# A Numerical Modeling of Formation of Volcanic Geothermal Reservoir

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***Abstract***

*The aim of the research is to develope the thermal energy from volcanic geothermal reservoir. One of the prospect of the geothermal energy for the future is the utilization of volcanic geothermal reservoir. The using of numerical simulation is important to clarify the thermal processes in a fluid reservoir beneath an active volcano. The fluid reservoir is a permeable zone, where the magmatic water discharged from magma mixes with the downgoing meteoric water. The flow rate of magmatic water becomes the mass and energy input to the reservoir. Finally, the fluid go to the earth's surface as vapor and liquid. The main physical parameters of the fluid reservoir cover pressure, temperature, enthalpy and enthalpy. The Finite Difference Method based on mass and heat balance equations is used to obtain these parameters. The parameters reperesent the thermal state of the fluid in the reservoir. The fluid phase in the reservoir changes with time. These processes interpret the development of the energy in a geothermal reservoir.*

***Keywords:*** *Geothermal reservoir, Numerical simulation, Permeable zone, Magmatic water, Thermal state, Finite difference method.*

## 1. Introduction

In the magmatic hydrothermal system, the magma is close to the bottom of the fluid reservoir[1). Faust and Mercer (1979) presented the geothermal reservoir simulation for liquid and vapor dominated[2). There are another simulation available, however most of them simulate the fluid condition for liquid and two phase state, in which the temperature is less or equal to the boiling point depth (BPD) temperature. The magma temperature is much higher than the BPD temperature. It means that in one occasion the higher temperature than BPD can be attained at the bottom area of the reservoir. In this research, a simulator is developed which may describe higher temperature than BPD. This aim is achieved by using Finite Difference Method base on the mass and heat balance equations. The results of the simulations is the physical state of fluid in the reservoir.

## 2. General Mathematical Model

The basis for general mathematical model of a geothermal system is the three dimensioanal flow of water and steam and heat transport. The movement of fluid through the permeable zone is assumed sufficiently slow. By this assumption, the Darcy's equation for multiphase flow may be used as simplified mementum balances. The Darcy's equation for fluid movement in porous media can be expressed as follows[2) :

$$Q = \frac{kk_{rw}}{\nu}(\nabla P - \rho g \nabla D) \quad (1a)$$

$$Q = \frac{kk_{rs}}{\nu}(\nabla P - \rho g \nabla D) \quad (1b)$$

Where $Q$ is the mass flux of fluid, $k$ is the intrinsic permeability of porous medium, $k_r$ is the relative permeability, $\nu$ is the kinematic viscosity, $P$ is the pressure, $\rho$ is the density of fluid, $g$ is the gravity acceleration and $D$ is the depth respectively. Subscripts $w$ and $s$ refer to liquid and steam state. There are two equations of general mathematical model are derived, i.e. mass balance and heat balance equation. The expression of the mass balance equation is as follows :

$$\frac{\partial}{\partial t}(\phi\rho_w S_w + \phi\rho_s S_s) + \nabla \bullet (Q_{mw} + Q_{ms}) - q_m = 0 \quad (2)$$

The heat balance equation can be expressed as follows:

$$\begin{aligned}&\frac{\partial}{\partial t}[\phi\rho_w S_w h_w + \phi\rho_s S_s h_s + (1-\phi)\rho_r h_r] \\ &+ \nabla \bullet (Q_{mw} h_w + Q_{ms} h_s) \\ &- \nabla \bullet [K(\frac{\partial T}{\partial P})_h \nabla P \\ &+ K(\frac{\partial T}{\partial h})_P \nabla h - q_e = 0\end{aligned} \quad (3)$$

In equation (2), $t$ is the time , $\phi$ is the porosity, $S_w$ is the water saturation, $S_s$ is the steam saturation, $Q_{mw}$ and $Q_{ms}$ are the mass flux of fluid for liquid and steam and $q_m$ is the mass source term, respectively. In equations (3), $h$ is the specific enthalpy, $T$ is the temperature, $P$ is the pressure, $K$ is the thermal conductivity of medium, $\rho_r$ is the rock density, $h_r$ is the rock enthalpy and $q_e$ is the energy source term respectively.

## 3. Physical Properties of Reservoir

The kinematic viscosity, density and thermal conductivity are the necessary properties of the fluid and reservoir to clarify the thermal processes in the fluid reservoir. Pressure and enthalpy as function of time are obtained by numerical simulation. The calculated saturates enthalpy ($h_s$) is obtained by known pressure using the regression formula as[3]:

$$h_s = 2822{,}82 - \frac{39{,}952}{P} + \frac{2{,}5434}{P^2} - 0{,}93888P^2 \tag{4}$$

The linearly interpolated values extracted from the Steam Table[4] is used to obtain the saturated enthalpy of water ($h_w$). The fluid phase at the time of interest may be established by knowing $h_s$ and $h_w$. If the computed values of pressure and enthalpy indicate the existence in a compressed water zone, then the density of water is one for $h \leq 200$, and for $h > 200$ the density is calculated by empiric formula as follows[3] :

$$\rho_w = 1{,}000 \times (0.98988 + 4.0089\times10^{-4} P - 4.0049\times10^{-5} h + \frac{2.6661}{h} + 5.4628\times10^{-7} Ph - 1.2996\times10^{-7} h^2) \tag{5}$$

If superheated steam is indicated, then

$$\rho_s = 1{,}000\times(-2.2616\times10^{-5} + 0.043844P - 1.7909\times10^{-5} Ph + 3.6928\times10^{-8} P^4 + 5.1764\times10^{-13} Ph^3) \tag{6}$$

In the above equations, the pressure, enthalpies and the densities are given in Mpa, kJ/kg and kg/m$^3$. The water saturation in the compressed water is one and in the superheated steam cindition is zero. Based on the calculated saturated enthalpies and densities, the water saturation is computed by using the following relations :

a. for $h \leq h_w$

$$S_w = 1.00 \tag{7a}$$

b. for $h_w \leq h \leq h_s$

$$S_w = \frac{\rho_s (h_s - h)}{h(\rho_w - \rho_s) - (h_w \rho_w - h_s \rho_s)} \tag{7b}$$

c. for $h \geq h_s$

$$S_w = 0 \tag{7c}$$

The bulk density ($\rho$) is obtained by :

$$\rho = S_w \rho_w + S_s \rho_s \tag{8}$$

The temperature for the compressed water zone and the superheated stean region are estimated by using the Steam Table[4]. The temperature of the two phase zone is calculated by :

$$T = \frac{4667.075}{12.599 - \ln(10P)} - 273.15 \tag{9}$$

Based on the calculated temperature, the fluid viscosities are obtained as below :

a. for liquid :

$$\upsilon_w = \frac{239.4\times10^{-7}\times10^{[248.37/(T+133)]}}{\rho_w} \tag{10}$$

b. for steam :

$$\upsilon_s = \frac{(0.407T + 80.4)\times10^{-7}}{\rho_s} \tag{11}$$

where $\upsilon_w$ and $\upsilon_s$ are given in m$^2$/s.

The relative permeability for liquid saturation is based on as follows[5] :

$$k_{rw} = \frac{(S_w - S_{wr})^4}{(1 - S_{wr} - S_{sr})^4} \tag{12}$$

and for steam is

$$k_{rs} = \left[1 - \frac{S_{wr}}{(1 - S_{wr} - S_{sr})}\right]^2 \times \left[1 - \frac{(S_w - S_{wr})^2}{(1 - S_{wr} - S_{sr})^2}\right] \tag{13}$$

In equations (12) and (13), $S_{wr}$ (=0.30) and $S_{sr}$ (=0.05) are residual water and steam saturation. In the balance equations, it is assumed that the capillary pressure effect are neglected. Rock enthalpy is a linear function of temperature as follows :

$$h_r = c_r T \tag{14}$$

where $c_r$ is the specific heat of the rock.

## 4. Numerical Solution

In this research, equations (2) and (3) are the main equations as the basis of the numerical simulator with their variables are pressure ($P$) and enthalpy ($h$). To obtain the values of pressure and temperature , eq. (1a) and eq. (1b) can be substituted to eq. (2), and in FDM form, it can be described as follows:

$$\frac{1}{(\Delta X_i)^2}\Delta[k_{ii}\lambda_w(\Delta P + \rho_w g \Delta D) + k_{ii}\lambda_s(\Delta P + \rho_s g \Delta D) + q_m = \frac{1}{\Delta t}\Delta[\phi\rho_w S_w + \phi\rho_s S_s) \tag{15}$$

In eq. (15), $k_{ii}$ is the permeability tensor in principal direction. Index $i$ = 1,2 and 3 refer to $x$, $y$, and $z$ axes, $\lambda_w = k_{rw}/v_w$ and $\lambda_s = k_{rs}/v_s$. Considering that $V_b$ is the grid block volume of the fluid (*Fig 1*) with $A$ as the sectional area perpendicular to the flow direction and $l_i$ is the length increment in the flow direction ($\Delta X_i$), then mass balances of eq (15) becomes:

$$\Delta\left[\begin{array}{l}\dfrac{k_{ii}\lambda_w A}{l_1}(\Delta P+\rho_w g\Delta D)\\ +\dfrac{k_{ii}\lambda_s A}{l_i}(\Delta P+\rho_s g\Delta D)\end{array}\right] + V_b q_m = \frac{V_b}{\Delta t}(M^{n+1}-M^n) \qquad (16)$$

where $M$ is the mass term ($M=\phi\rho_w S_w+\phi\rho_s S_s$).

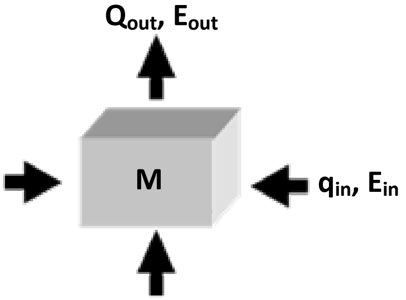



Figure 1. A block volume of the fluid reservoir

Similarly, by substituting eq. (1a) and (1b) to eq. (3), for heat balance in the FDM form can be written as follows[6] :

$$\begin{aligned}\frac{1}{(\Delta X_i)^2}\Delta[&k_{ii}\lambda_w h_w(\Delta P+\rho_w g\Delta D)\\ &+k_{ii}\lambda_s h_s(\Delta P+\rho_s g\Delta D)\\ &+K(\frac{\partial T}{\partial P})_h\nabla P+K(\frac{\partial T}{\partial h})_P\nabla h]+q_e\\ =\frac{1}{\Delta t}\Delta[&\phi\rho_w S_w h_w+\phi\rho_s S_s h_s\\ &+(1-\phi)\rho_r h_r\end{aligned} \qquad (17)$$

where H is enthalpy of fluid respectively.

In a block volume of $V_b$, eq. (17) can be expressed as :

$$\Delta\left[\begin{array}{l}\dfrac{k_{ii}\lambda_w A}{l_1}(\Delta P+\rho_w g\Delta D)\\ +\dfrac{k_{ii}\lambda_s A}{l_i}(\Delta P+\rho_s g\Delta D)\end{array}\right] + V_b q_m = \frac{V_b}{\Delta t}(M^{n+1}-M^n) \qquad (18)$$

Furthermore, by the Taylor expansion, a set of linear equation in $\delta P$ and $\delta h$ can be obtained to solve eq. (16) and (17) numerically as follows[7]:

a. for mass balance :

$$\phi c_w\rho_w\frac{P^{n+1}_{i,j}-P^n_{i,j}}{\Delta t}-\frac{k}{\upsilon}\left(\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)-\frac{k}{\upsilon}\left(\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)-q_{in}=0 \qquad (19)$$

By Crank Niholson technique, the mass balance can be expressed as:

$$\phi c_w\rho_w\frac{P^{n+1}_{i,j}-P^n_{i,j}}{\Delta t}-\frac{k}{2\upsilon}\left(\frac{P^{n+1}_{i+1,j}-2P^{n+1}_{i,j}+P^{n+1}_{i-1,j}}{\Delta x^2}+\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)$$
$$-\frac{k}{2\upsilon}\left(\frac{P^{n+1}_{i,j+1}-2P^{n+1}_{i,j}+P^{n+1}_{i,j-1}}{\Delta z^2}+\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)-q_{in}=0 \qquad (20)$$

or

$$P^{n+1}_{i,j}=P^n_{i,j}+\frac{k\Delta t}{2\upsilon\phi c_w\rho_w}\left(\frac{P^{n+1}_{i+1,j}-2P^{n+1}_{i,j}+P^{n+1}_{i-1,j}}{\Delta x^2}+\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)$$
$$+\frac{k\Delta t}{2\upsilon\phi c_w\rho_w}\left(\frac{P^{n+1}_{i,j+1}-2P^{n+1}_{i,j}+P^{n+1}_{i,j-1}}{\Delta z^2}+\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)+\frac{q_{in}\Delta t}{\phi c_w\rho_w} \qquad (21)$$

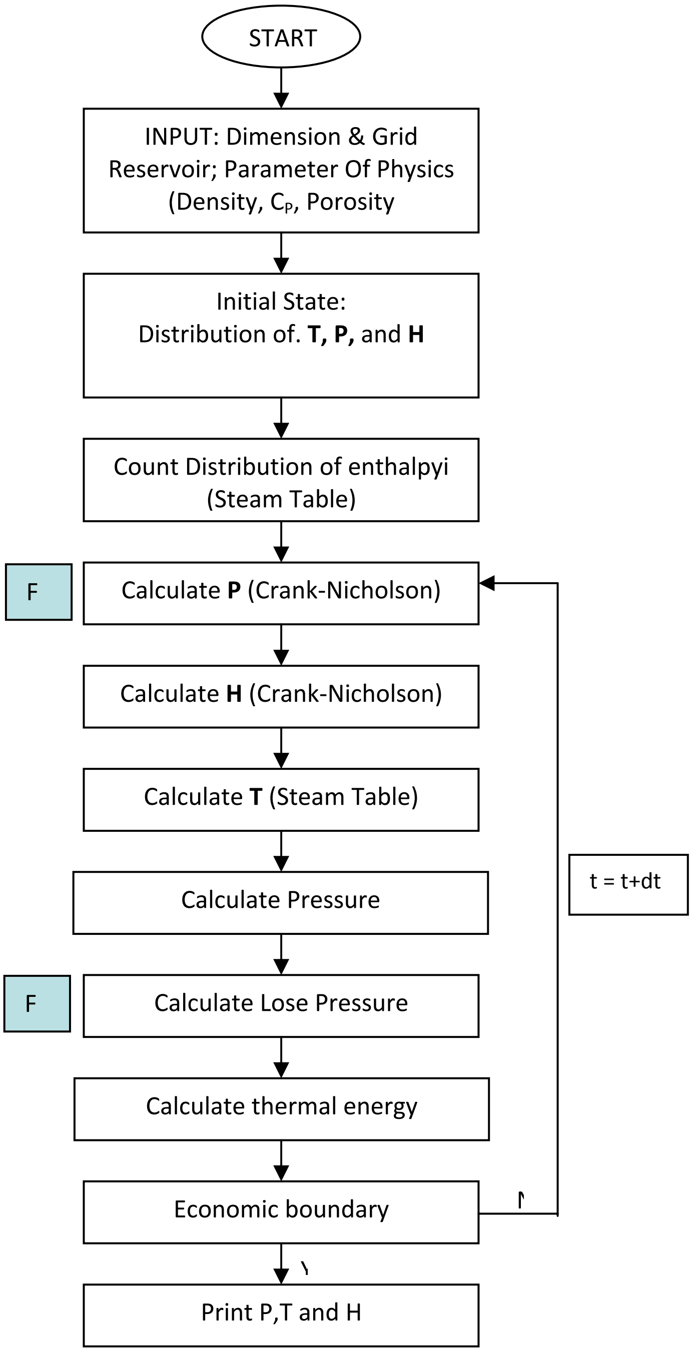
b. for heat balance:

$$\left[(1-\phi)\rho_m+\phi\rho_w\right]\frac{\left(H^{n+1}_{i,j}-H^n_{i,j}\right)}{\Delta t}-$$
$$\frac{k}{\upsilon_w}\left(\frac{H^n_{i+1,j}-H^n_{i,j}}{\Delta x}\right)\left(\frac{P^n_{i+1,j}-P^n_{i,j}}{\Delta x}\right)-\frac{kH^n_{i,j}}{\upsilon_w}\left(\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)-$$
$$\frac{k}{\upsilon_w}\left(\frac{H^n_{i,j+1}-H^n_{i,j}}{\Delta z}\right)\left(\frac{P^n_{i,j+1}-P^n_{i,j}}{\Delta z}\right)-\frac{kH^n_{i,j}}{\upsilon_w}\left(\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)-$$
$$K\mu_{JT}\left(\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)-\frac{K}{C_P}\left(\frac{H^n_{i+1,j}-2H^n_{i,j}+H^n_{i-1,j}}{\Delta x^2}\right)-$$
$$K\mu_{JT}\left(\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)-\frac{K}{C_P}\left(\frac{H^n_{i,j+1}-2H^n_{i,j}+H^n_{i,j-1}}{\Delta z^2}\right)-E'''_{in}=0 \qquad (22)$$

Similarly, by Crank Niholson technique, the mass balance can be expressed as:

$$((1-\phi)\rho_m+\phi\rho_w)\frac{H^{n+1}_{i,j}-H^n_{i,j}}{\Delta t}-\frac{k}{2\upsilon}\left(\frac{P^n_{i+1,j}-P^n_{i,j}}{\Delta x}\right)\left(\frac{H^{n+1}_{i+1,j}-H^{n+1}_{i-1,j}}{\Delta x}+\frac{H^n_{i+1,j}-H^n_{i-1,j}}{\Delta x}\right)$$
$$-\frac{k}{\upsilon}\left(\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)\left(\frac{H^{n+1}_{i,j}+H^n_{i,j}}{2}\right)-\frac{k}{2\upsilon}\left(\frac{P^n_{i,j+1}-P^n_{i,j}}{\Delta z}\right)\left(\frac{H^{n+1}_{i,j+1}-H^{n+1}_{i,j-1}}{\Delta z}+\frac{H^n_{i,j+1}-H^n_{i,j-1}}{\Delta z}\right)$$
$$-\frac{k}{\upsilon}\left(\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)\left(\frac{H^{n+1}_{i,j}+H^n_{i,j}}{2}\right)-K\mu_{JT}\left(\frac{P^n_{i+1,j}-2P^n_{i,j}+P^n_{i-1,j}}{\Delta x^2}\right)$$
$$-K\mu_{JT}\left(\frac{P^n_{i,j+1}-2P^n_{i,j}+P^n_{i,j-1}}{\Delta z^2}\right)-\frac{K}{2C_P}\left(\frac{H^{n+1}_{i+1,j}-2H^{n+1}_{i,j}+H^{n+1}_{i-1,j}}{\Delta x^2}+\frac{H^n_{i+1,j}-2H^n_{i,j}+H^n_{i-1,j}}{\Delta x^2}\right)$$
$$-\frac{K}{2C_P}\left(\frac{H^{n+1}_{i,j+1}-2H^{n+1}_{i,j}+H^{n+1}_{i,j-1}}{\Delta z^2}+\frac{H^n_{i,j+1}-2H^n_{i,j}+H^n_{i,j-1}}{\Delta z^2}\right)-E'''_{in}=0 \qquad (23)$$

A part of the flow chart of the program is as follows:




## 5. Results and Discussion

In this numerical sumulation model, 30 kg/m of magmatic water as energy comes from magma with its enthapy about 3500 kJ/kg. The downgoing meteoric water comes from the earth's surface and then it enters to the central part of the reservoir. Figure 2, Figure 3, Figure 4 and Figure 5 show the fluid temperature in reservoir from the initial time to 50 years. The high temperature is at the bottom of the reservoir. It can be seen that the state of fluid is in two phase at arround of central part of the reservoir, especially at the bottom of the central of reservoir. The temperature at the top of the central of reservoir is about 200$^{o}$ C. The development of temperature to be decrease after 50 years. The enthalpies distribution pattern almost have same type with the temperature distribution (see Fig 5, Figure 6, Fig 7 and Fig 8). This means that it is possible to exstract energy from the reservoir. This results can be used to estimate the heat discharged rate from reservoir as preliminary study about geothermal reservoir energy prospect. The gethermal energy increasing of the system is the heat supplied to the system minus the work done by it which can be expressed as follows[8] :

$$U_2 - U_1 = L - P(V_2 - V_1) \qquad (24)$$

and

$$L = H_2 - H_1 \qquad (25)$$

In equations (24) and (25), $U$ is the internal energy, $V$ is the volume $L$ is quantity of the latents of transformation, and $H$ is the enthalpy respectively. The total mass output at the is about 51 kg/s. This result means that 40 kg/s of magmatic water mixes with 11 kg/s of the downgoing meteorc water. This means that about 80 % of the discharged water at the surface is in magmatic water origin[9]. This is a typical feature of magmatic hydrothermal system, where the downgoing meteoric water comes from earth's surface and then it enters to the reservoir. Furthermore, in an aplication, the output of liquid and steam part at the surface can be calculated. Finally the heat discharged rate can be obtained by using the equations.

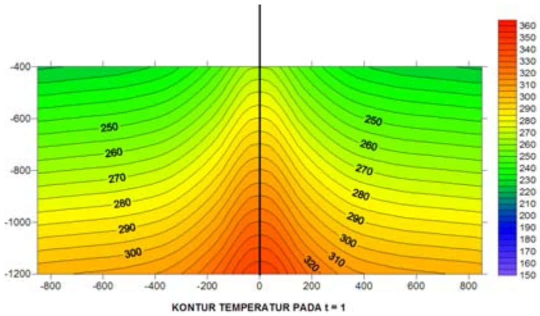



Figure 2. Temperature Distribution contour at time t =1 year

During a change of state at constant pressure, the decrease or increase of enthalpy is equal to the Latent heat transformation.

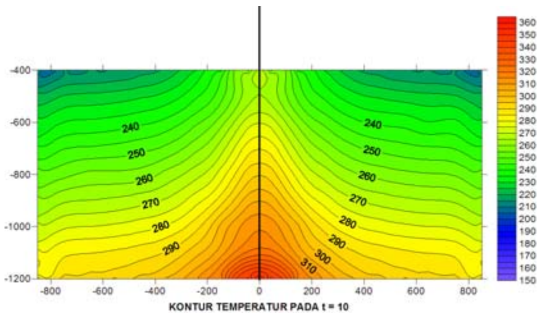



Figure 3. Temperature Distribution contour at time t = 10 years

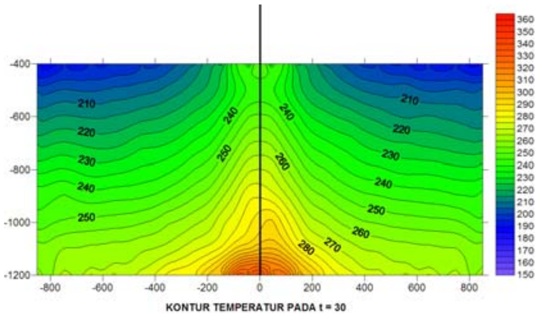



Figure 4. Temperature Distribution contour at time t =30 years

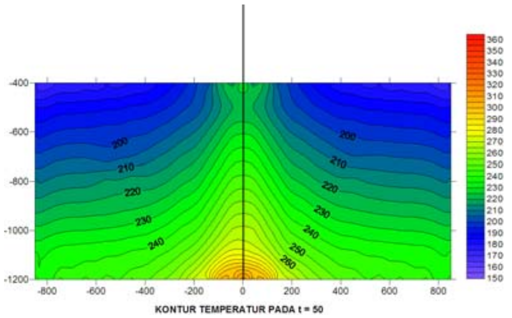



Figure 5. Temperature Distribution contour at time t =50 years

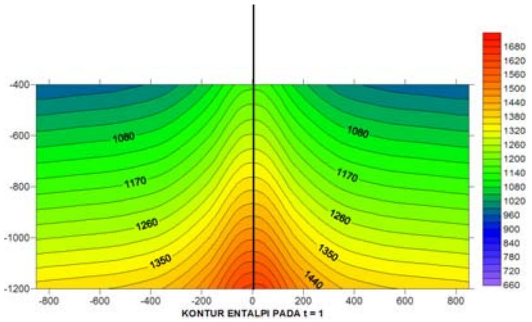



Figure 6 Distrubution of enthalpies contour at time t = 1 year

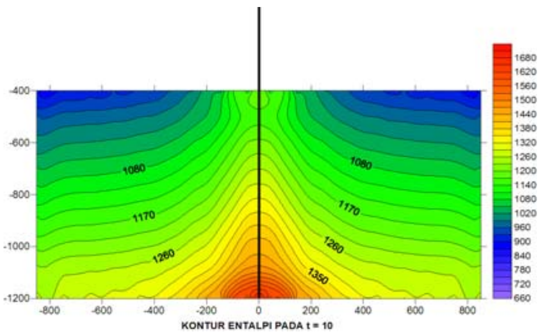



Figure 7. Distrubution of enthalpies contourat time t = 10 years

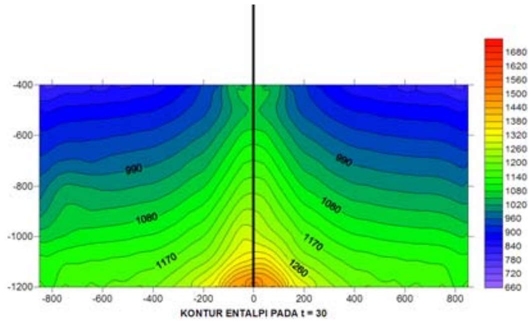



Figure 8. Distrubution of enthalpies contour at time t = 30 years

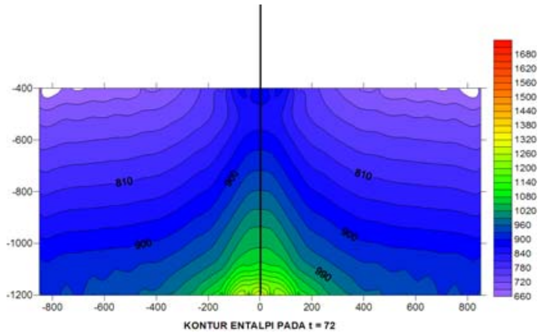



Figure 9. Distrubution of enthalpies contour at time t = 72 years

## 6. Conclusion

In this study, a numerical modeling of formation of thermal processes of volcanic geothermal reservoir is developed. This work is done as an effort to simulate the fluid energy state at the reservoir. As a result, the main physical parameters of the fluid reservoir cover pressure, temperature and enthalpy. These parameters represent the thermal state of the fluid in the reservoir as a function of time. The estimated values of the parameters are clarified by using the Finite Difference Method based on the mass and heat balance equations. The numerical modeling results show the fluid temperature in reservoir from the initial time to several years. The high temperature is shown at around of central of the reservoir due to two phase state fluid in this area. Based on this result and other supporting geophysical data, the geothermal energy can be exploited from the reservoir.

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