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Performance Analysis of 2D and 3D Fluid Flow Modelling Using Lattice Boltzmann Method

Fourier Dzar Eljabbar Latief and Umar Fauzi
Physics of Complex System Divison,
Faculty of Mathematic and Natural Sciences
Institut Teknologi Bandung, Indonesia
e-mail: dzareljabbar@students.itb.ac.id; umarf@fi.itb.ac.id

Abstract

Several studies have been conducted to observe properties of fluid flow in materials using the Lattice Boltzman Method (LBM). There are two widely used lattice model, the D2Q9 for the 2D simulation, and the D3Q19 for the 3D simulation. Our particular interest is to study the velocity map both using the 2D and the 3D simulation, using the same object.

The aim of this study is to evaluate effectiveness and efficiency of both methods. In our simulation, the velocity profile between the 2D and 3D models differs greatly (mean error 30.4%) if the object has complex lateral structure (the shape along the z-axes differs greatly), while for the less complex object, the profile has only 1.4% of mean error. The computing time for the 3D model took 13 times longer than the simulation of the 2D model. The result from the comparison of both methods concludes that the simplification of fluid flow simulation of 3D objects into 2D objects should be taken carefully, for in some cases, the simplification is not quite appropriate.

Keywords : *Fluid Flow Simulation, LBM, D2Q9, D3Q19*

1. Introduction

In recent years, Lattice Boltzmann Method (LBM) has become increasingly popular due to their ease of implementation, extensibility, and computational efficiency. Consequently, Lattice Boltzmann has become a viable alternative to traditional Computational Fluid Dynamic (CFD) methods. The Lattice Boltzmann (LB) approach, has been increasingly used in various engineering applications in modeling the flow of both single and multi-component fluids[1-6].

The main advantages of the Lattice Boltzmann Method include easy implementation of boundary conditions and computational efficiency by allowing parallel computing. The method also accommodates boundary conditions such as a pressure drop across the interface between two fluids and wetting effects at the fluid-solid interface[7]. It has proven to be very accurate in simulating isothermal, incompressible flow at low Reynolds numbers[5].

2. Lattice Boltzmann Method in Modeling Fluid Flow

Among various techniques in fluid flow modeling, Lattice Boltzmann Method has been gaining wide acceptance due to its ease of implementation of boundary conditions and numerical stability in wide variety of flow conditions with various Reynolds numbers. It has evolved from the Lattice Gas Automata (LGA)[8]. Various difficulties experienced in LGA were overcome by the introduction of the LBM. It was first introduced by McNamara and Zanetti[9] to eliminate the statistical noise in the LGA. Since then, it has been implemented and improved by various researchers in variety of disciplines. Early applications of the Lattice Boltzmann Method to porous media largely focused on the feasibility of the method. Succi *et al.*[5] used LBM to simulate flow through random pack of blocks and demonstrated the adherence to Darcy's law. Various researchers[10-13] improved the LBM by introducing a variety of new boundary conditions for the solid boundaries.

Maier *et al.*[14] implemented a three dimensional LB model (D3Q19) to simulate flow through bead packs and compared with the Kozeny-Carman prediction for the sphere packing. Kim[15] utilized a two dimensional (D2Q9) LB model to simulate flow through rock fractures and compared with the analytical equations that assumes the rock fracture composed of set of parallel plates. Martys *et al.*[16] successfully applied LBM to simulate a multiphase flow through Fontainebleau 7 sandstone. Hornero *et al.*[17] measured the performance of a two dimensional LB model for simulating soil flow in a simple erosion model and compared the results to those predicted by an analytical solution. Tang *et al.,*[18] successfully implemented LBM to simulate gas flow through microchannels.

Flow through fibrous materials such as papers, random fiber webs and woven fabrics have been the interest of researchers using LBM. Koponen *et al.,*[19] modeled the hydraulic conductivity of three-dimensional random fiber webs using LBM and found a good agreement with the experimental measurements. Filippova[20] used a three-dimensional LBM to model gas-particle flow through filters and successfully simulated filtration phenomenon. Clague[21)] modeled hydraulic conductivity of bounded and unbounded fibrous media using the LBM.

In the LBM, fluid is considered as a collection of particles that are represented by a particle velocity distribution function at each discrete lattice node. Particles collide with each other and properties associated with the lattice notes are updated at discrete time steps. The rules governing the collisions are designed such that the time-average motion of the particles is consistent with the Navier-Stokes equations. In LBM, the computation of each node at every time step depends solely on the properties of itself and the neighboring nodes at the previous time step.

Various LB models exist for numerical solution of various fluid flow scenarios, where each model has different way of characterizing microscopic movement of the fluid particles. The LB models are usually denoted as DxQy where x and y corresponds to the number of dimensions and number of microscopic velocity directions (e_i), respectively (Table 1). For example, D2Q9 represents a two-dimensional geometry with nine microscopic velocity directions. The following sub chapter will give general picture of the lattice D2Q9 and D3Q19 LB models which were implemented in this study.

These models obey the distributions function stated as in Kutay[22)]:

$$F_i^{eq} = w_a\,\rho\left[1+\frac{\mathbf{e}_i\cdot\mathbf{u}}{c_s^2}+\frac{(\mathbf{e}_i\cdot\mathbf{u})^2}{2c_s^4}-\frac{(\mathbf{u}\cdot\mathbf{u})^2}{2c_s^2}\right] \quad (1)$$

where F_i^{eq} is the equilibrium distribution function, ρ is the density, **u** is the macroscopic velocity of the node. The relaxation time relates to viscosity of fluid (ν) as follows:

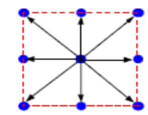
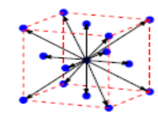
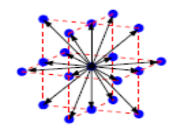
$$\upsilon = c_s^2(\tau-\tfrac{1}{2}) \quad (2)$$

The macroscopic properties, density and velocity, of the nodes are calculated using the following relations:

$$\rho=\sum_{i=1}^{Q}F_i\;,\;\mathbf{u}=\frac{\sum_{i=1}^{Q}F_i}{\rho} \quad (3)$$

where ρ and **u** are the macroscopic density and velocity of the fluid each node of lattice.

Table 1: Properties of various LB models

Model Name	Lattice Speed of Sound, (c_s^2)	Weight Factors	Velocity Directions
D1Q3	1/3	4/6 (1) 1/6 (2)	
D1Q5	1	6/12 (1) 2/12 (2) 1/12 (2)	
D2Q9	1/3	16/36 (1) 4/36 (2) 1/36 (3)	
D3Q15	1/3	16/72 (1) 8/72 (2) 1/72 (3)	
D3Q19	1/3	12/36 (1) 2/36 (2) 1/36 (3)	

Note: Weight factors for; (1) rest particle, (2) face-connected neighbors, (3) edge-connected neighbors.

2.1 The 2D fluid flow modeling

A very common lattice for 2D isothermal fluids is the D2Q9 lattice that connects the lattice sites only with their nearest neighbors. The D2Q9 model shown in Figure 1 is a two-dimensional model (D2) with nine possible velocity vectors (Q9). At a given time step, the center particle may travel to any of the eight surrounding nodes, or it may remain stationary. Thus, a velocity vector equal to zero constitutes the ninth possible vector. Visibly, the space of velocities has obviously been reduced dramatically, as it is left with only 9 elements. It can however be shown that this is sufficient for several purposes. The number of lattice velocities must in general be chosen in such a way as to respect some specific isotropy relations, so that the dynamics of the fluid is asymptotically isotropic on high resolution lattices.

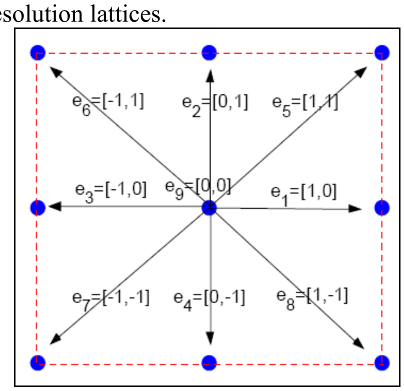


Figure 1. Lattice model for D2Q9.

2.2 The 3D fluid flow modeling

As has been explained before, Lattice Boltzmann can also be used to simulate three-dimensional flows with models such as the D3Q19, which has motion in three dimensions and 19 associated velocity vectors.

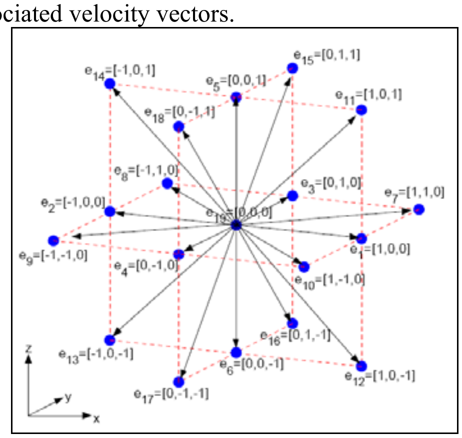


Figure 2. Lattice model for D3Q19.

2.3 Velocity map comparison between 2D and 3D

In many real cases, fluid flow in 3D objects can often be simplified in to the 2D projection along the interested plane. But in some other certain cases, the result shows that such simplification is not appropriate to be taken. Here we present examples on cases which such simplification should not be made.

First we used a simple model of 3D object obstacle as can be seen in Figure 3. The simulation of the fluid flow is then made using the D3Q19 model to observe the velocity pattern.

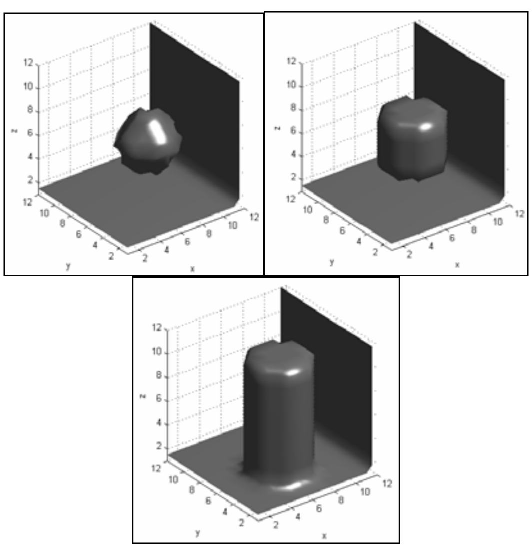


Figure 3. Three dimensional obstacles that are applied to map the velocity pattern.

The result of this simulation is shown in Figure 4. The slice on the X-Y plane at Z = 7 is then observed. The 3D obstacles from Figure 3 are sliced at the same Z point. The results of these slices are used as 2D obstacles model to simulate the fluid flow using the D2Q9 lattice model. In both lattice models (D3Q19, D2Q9) the fluid flow directed along the X-Y plane, parallel to the Y axes. For this comparison, we calculated mean errors based on the following formula:

$$Err = \sum_{i,j}^{m,n} \left(\frac{\left| v_{i,j}^{3D} - v_{i,j}^{2D} \right|}{v_{i,j}^{3D}} \right) \frac{1}{m \cdot n} \qquad (4)$$

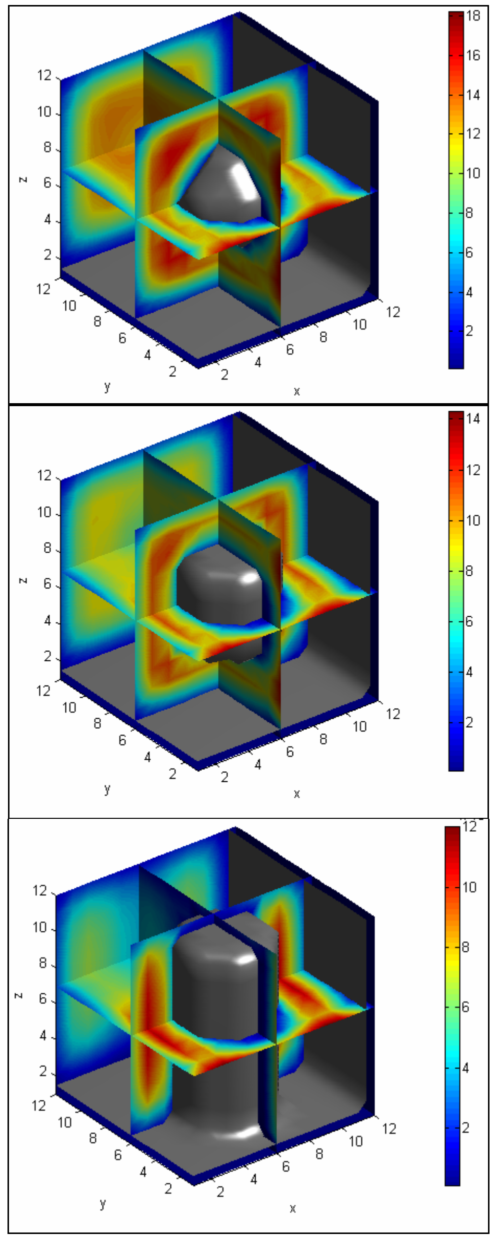


Figure 4. Slice of the velocity pattern from each of the 3D models.

The result from the first 3D model slices is shown in Figure 5, as we can see, the pattern is quite different. We calculated the mean error from the difference and the result is ±30.4%.

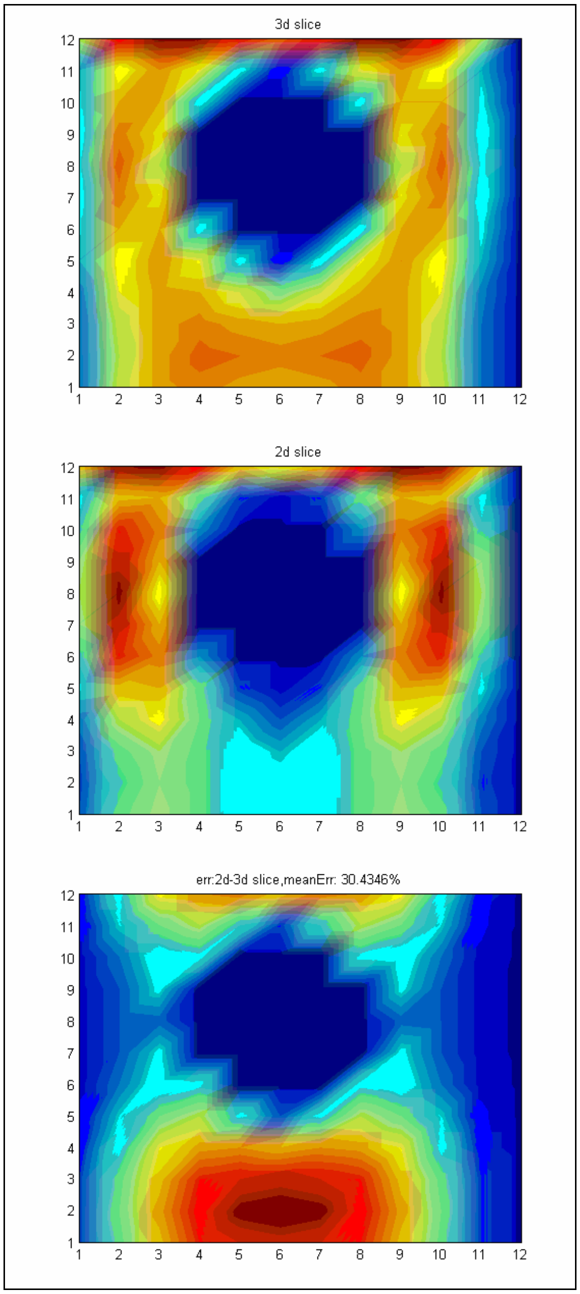


Figure 5. Result from the first model.

The result from the second 3D model slices can be seen in Figure 6. The pattern is quite different. We calculated the mean error from the difference and the result is ±9.2%.

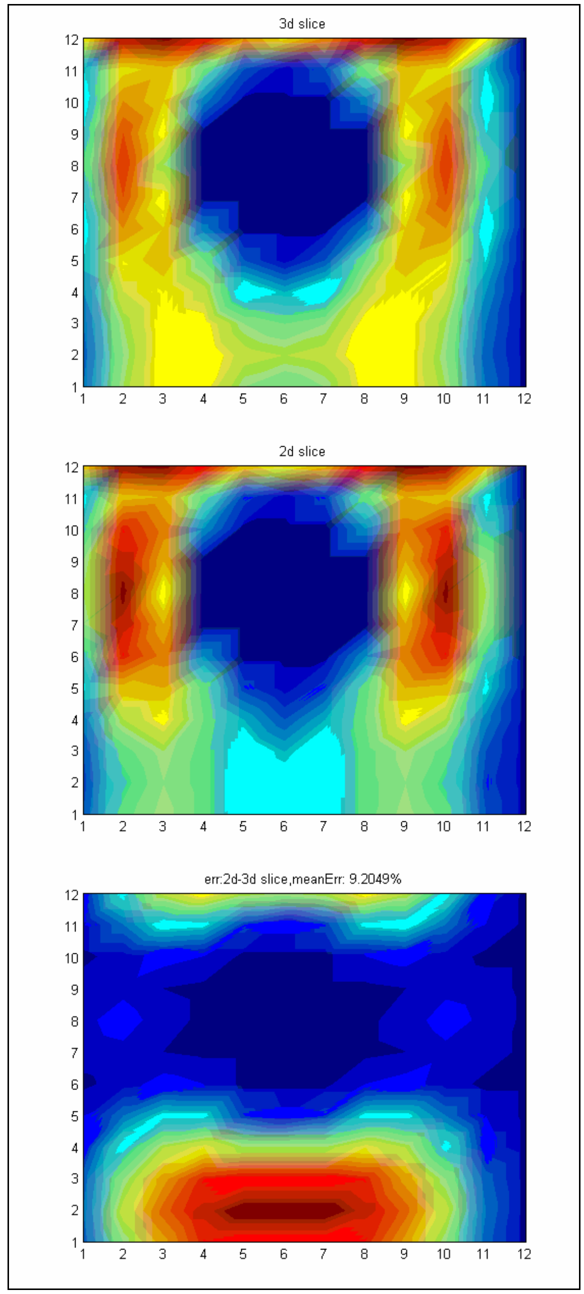


Figure 6. Result from the second model.

The result from the third 3D model slices can be seen in Figure 7. We calculated the mean error from the difference and the result is ±1.4%.

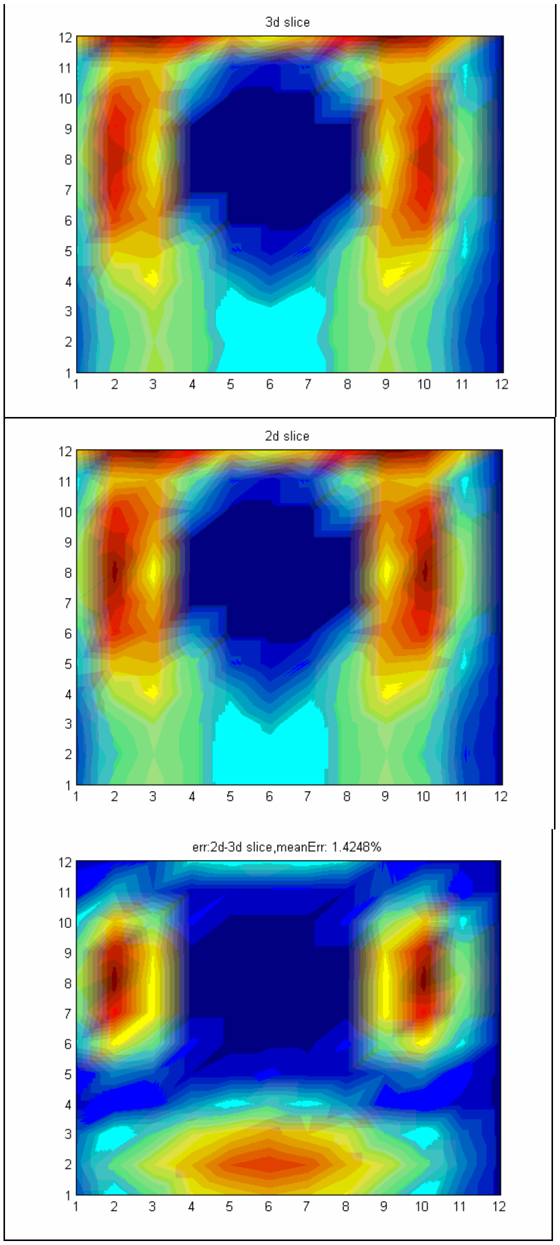


Figure 7. Result from the third model.

As we can see from the results, the more similar the obstacles structure (in perpendicular to fluid flow direction) the smaller the error is.

2.4 Comparison of computing time

One of many interesting issue in CFD using LBM is the computing time. For large-scale objects, the difference of the computing time between the 2D and the 3D is very significant. We calculate the computing time using a larger scale (120x60x5 pixel) complex model (Figure 8).

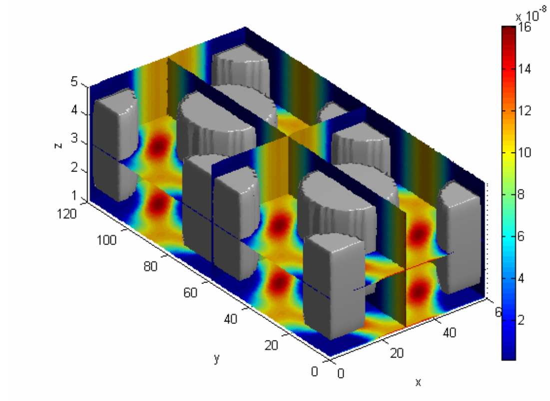


Figure 8. Model used to calculate the computing time.

In the computing time for the 2D model is 25.9380 seconds. While computing time for the 3D model is 330.9840 seconds, which is more than 13 times longer. The algorithm was executed in an AMD Athlon™ 64 X2 Dual Core Processor 4600+ 2.41 GHz, 2GB of RAM using Matlab v.7.

The 3D algorithm has a serious computing time issue. Fortunately, this disadvantage might be overcome by modifying the algorithm. Various modifications have been conducted by many researches. Wellein, *et al.*,[23)] utilized a so-called *memory hierarchy* to optimize the 3D LBM.

3. Discussion

In the comparisons above, the slices were taken in parallel with the flow of the fluid. As we observed, the 2D and 3D maps are much different when the lateral (direction along the z-axes) structure of the obstacle (pore structure) also differs much. In the first model, the third velocity component of the 3D fluid flow (along the z-axes) has significant contribution to overall velocity scalar value (map). While in the second and the third model, the contribution decreases, as the lateral structure become more similar.

4. Conclusions

From the discussion above, we could conclude that the 2D lattice model could not map the velocity profile of three dimensional object appropriately in the case that the lateral structure varies much and the fluid flow is along the horizontal plane. Thus, in order to obtain precise model of the fluid flow in three dimensional complex structure objects (e.g., hydrocarbon reservoir), it is important to simulate it using the 3D lattice model approach. Such simplification from 3D to 2D is appropriate in cases where the obstacle structure does not vary in such complex way, or in some cases that such variations are negligible, for example, in case of river flow velocity mapping. In this perspective, the simplification can reduce the large computing time performed by the 3D model.

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