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M Theory Compactified On G_2 Manifold With Background Fluxes

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Abstract

Some geometrical aspects of M-theory compactified on seven dimensional G_2 manifold with background fluxes are considered. It turns out that this model admits N=1 supersymmetry. We also discuss the arising holomorphic superpotential and the scalar potential with its possible vacua.

Keywords: *Supersymmetry, Compactification, G_2 manifold*

1. Introduction

An eleven-dimensional M-theory[1)] has became a prominent theory during the last ten years for both theorist and phenomenologist (see for example[2,3)]). This theory is believed to be the fundamental picture of the known string theories in ten dimensions, *i.e.* $SO(32)$ Heterotic, $E_8\times E_8$ Heterotic, Type IIA, Type IIB and Type I, which might also be a candidate for unification theory in four dimensional physics.

The main interest until now is how to construct M-theory vacua which have four macroscopic spacetime dimensions and a realistic particle spectrum. Since M-theory itself is intrinsically supersymmetric, it might be more natural to consider its supersymmetric vacua in four dimensions. Since extended $N \geq 2$ supersymmetry in four dimensions cannot accommodate chirality of the standard model, we should study M-theory vacua with N=1 supersymmetry. Thus to find our demanding vacua, M-theory must be compactified on a seven dimensional manifold X^7 whose its holonomy is G_2 [4)].

If X^7 is large compared to the Planck scale and smooth, then at low energy this M-theory is described by eleven dimensional supergravity. Compactifications of this eleven dimensional supergravity have been considered for several decades (see for a review[4)]). In the last few years there has been a tremendeous progress in studying M-theory on singular G2 manifold to obtain chiral fermions which is the basic requisites of the standard model[5,6)].

In this paper we study the low energy limit of M-theory which is provided by eleven dimensional supergravity on general G2 manifold in the presence of fluxes. We apply our study to cosmological model of early universe based on local supersymmetry breaking. This paper is organized as follow. The next section is devoted to introduce M-theory and its relation with string theories. G2 manifold with background fluxes is discussed in section 3. Supersymmetry breaking and its application to the early universe is studied in section 4. We summarize our result and discuss our future work in section 5.

2. Evidence of M-Theory ?

As we have mentioned in the previous section, M-theory is an eleven-dimensional theory which is conjectured to be the fundamental theory describing the five consistent string theories. Here we only discuss this story using two situations, *i.e.* the type IIA and the $E_8\times E_8$ heterotic cases and leave some facts below ten dimensions for the interested readers in the reference[1)]. It is well known that the low energy effective action of the type IIA string is provided by the type IIA supergravity in ten dimensions which can be constructed as S^1 compactification of the eleven dimensional supergravity. This implies a relation between the radius R_{11} of the eleventh dimension and the string coupling constant $g_s = e^{\langle\phi\rangle}$, where ϕ is the dilaton field

$$R_{11} = \frac{L}{2\pi} g_s^{\frac{3}{2}} , \tag{1}$$

where L is the length of the eleventh dimension. Furthermore, the Kaluza-Klein (KK) spectrum of this theory obeys

$$M^{\mathrm{KK}} = \frac{2\pi |n|}{g_s\, L}, \tag{2}$$

where n is an arbitrary integer. These KK-states do not belong to the perturbative type IIA spectrum because they become heavy in the weak coupling limit $g_s \to 0$. However, in the strong coupling limit $g_s \to \infty$ they become light and can no longer be neglected in the effective theory. This infinite number of light states (which is called D-particles of type IIA theory) indicates that the theory effectively decompactifies. Supersymmetry is unbroken in this limit and thus the KK-states assemble in supermultiplets of the eleven-dimensional supergravity. Since there is no string theory which has eleven-dimensional supergravity as the low energy limit, the strong coupling limit of type IIA string has to be a new theory, called M-theory.

Another surprising result occurs in the strong coupling limit of the heterotic $E_8 \times E_8$ theory. This theory can be constructed by compactifying M-theory on a $\boldsymbol{Z}_2$ orbifold of S^1 [1]. Just as in the type IIA case one has $R_{11} = g_{\mathrm{H}}^{3/2}\, L/2\pi$ and thus weak coupling corresponds to small R_{11} and the two ten-dimensional hyperplanes sit close to each other; in the strong coupling limit the two ten-dimensional hyperplanes move far apart (to infinity). Thus the heterotic $E_8 \times E_8$ string theory can be viewed as M-theory compactified on $S^1/\boldsymbol{Z}_2$.

The above examples can be used to establish the statement that there is only one underlying theory, *i.e.* M-theory whose moduli space (or the manifold of ground states) embraces all string theories[1], which is schematically shown in Figure 1. At certain corners of the moduli space it looks effectively ten-dimensional and can be described by a weakly coupled string theory. Its low energy limit is given by eleven-dimensional supergravity but its precise non-perturbative formulation still has to be found.

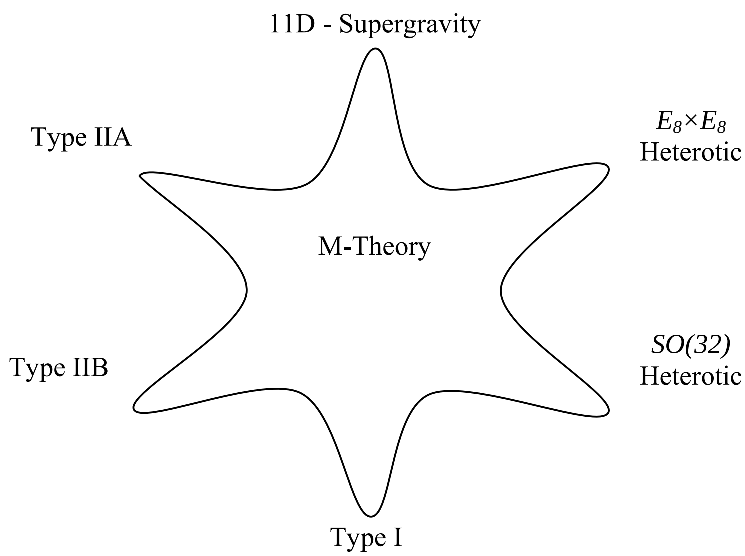

Figure 1. M-theory picture of string theory

3. Low Energy Effective M Theory

In this section we will shortly discuss the eleven-dimensional supergravity theory as the low energy limit of M-theory which was first constructed in the reference[7]. The emerging of G_2-holonomy manifolds in compactification of M-theory will then be described. Finally, we will discuss its compactification on G_2-holonomy manifolds with background fluxes.

Supergravity in eleven dimensions consists of an "elfbein" field E_M^A, a gravitino field Ψ_M, and a three-form field C_{MNP}, where $A = 0,...,10$ are the flat indices while $M, N, P = 0,...,10$ are the curved indices. The Lagrangian has the form in the following[7]:

$$L_{11} = \frac{1}{\kappa_{11}^2} E\left[-\frac{1}{2} R(E,\Omega) - \frac{1}{2} \Psi_M\, \Gamma^{MNP} D_N \Psi_P - \frac{1}{48} G_{MNPQ}\, G^{MNPQ} \right]$$
$$\frac{1}{3456\, \kappa_{11}^2} \sqrt{2}\, \varepsilon^{MNPQRSTUVWX}\, G_{MNPQ}\, G_{RSTU}\, C_{VWX}$$
$$-\frac{1}{192\, \kappa_{11}^2} E\left(\overline{\Psi}_R\, \Gamma^{MNPQRS}\, \Psi_S + 12\, \overline{\Psi}^M\, \Gamma^{NP}\, \Psi^Q \right) G_{MNPQ} + ... \tag{3}$$

where the triple dots denote terms of order Ψ^4 and higher, $E = \det E_M^A$ and $\Omega_M{}^{AB}$ denotes the spin connection. An antisymmetric quantity $G_{MNPQ} = 24\, \partial_{[M} C_{NPQ]}$ are called fluxes (or G-fluxes). The supersymmetry variations of fields are defined as

$$\delta E_M^A = \frac{1}{2} \bar{\varepsilon} \Gamma^A\, \Psi_M \tag{4}$$

$$\delta C_{MNP} = -\frac{1}{8}\sqrt{2}\,\bar{\varepsilon}\,\Gamma_{[MN}\Psi_{P]} \tag{5}$$

$$\delta\Psi_M = D_M(\hat{\Omega})\varepsilon + \frac{1}{288}\sqrt{2}\left(\Gamma_M{}^{NPQR} - 8\,\delta_M^N\,\Gamma^{PQR}\right)\varepsilon\,\hat{G}_{NPQR} \tag{6}$$

Here the covariant derivative is defined by

$$D_M\left(\hat{\Omega}\right)\varepsilon = \left(\partial_M - \frac{1}{4}\hat{\Omega}_M{}^{AB}\,\Gamma_{AB}\right)\varepsilon \tag{7}$$

where $\hat{\Omega}_M{}^{AB} = \Omega_M{}^{AB} + \frac{\kappa_{11}}{8}\sqrt{2}\,\overline{\Psi}_N\,\Gamma_M{}^{ABNP}\,\Psi_P$ and $\hat{G}_{MNPQ}$ is the supercovariant field strength

$$\hat{G}_{MNPQ} = G_{MNPQ} + \frac{3}{2}\sqrt{2}\,\Psi_{[M}\Gamma_{NP}\Psi_{Q]} \tag{8}$$

Note the presence of the Chern-Simons-like terms in the Lagrangian, *i.e.* the fourth term in (3), that leads the action is only invariant up to surface terms. We want to mention that the quartic-Ψ terms can be included into the Lagrangian (3) by replacing the spin-connection field Ω by $\left(\Omega+\hat{\Omega}\right)/2$ in the covariant derivative of the gravitino kinetic term and by replacing G_{MNPQ} in the last line by $(G_{MNPQ}+\hat{G}_{MNPQ})/2$. These subtitutions ensure that the field equations corresponding to (3) are supercovariant.

4. 11D Supergravity on G_2 Manifold with Fluxes

Next we discuss about compactification and how to obtain N=1 configuration of M-theory vacua which preserves fully Lorentz invariant in four dimensions. The basic assumption of compactification is to consider that some of the eleven dimensions are actually small and compact and only four are extended and can be observed. Consequently one then chooses the eleven dimensional space to be a direct product of a four dimensional (we choose here the flat Minkowski space) and an internal unknown manifold X^7

$$\mathrm{M}^{1,10} = \mathrm{R}^{1,3}\times X^7$$

This is equivalent to choosing a background metric which is a direct product between the four dimensional Minkowski metric and the metric on the internal manifold X^7. In order to have N=1 vacua in four dimensions one has to set vanishing the vacuum expectation value of the supersmmetry variation of Ψ_M for $G_{MNPQ}=0$ (restricted) in the seven dimensional compact manifold X^7 :

$$\left\langle\delta\Psi_I\right\rangle = D_I\left(\left\langle\Omega\right\rangle\right)\varepsilon = 0 \tag{9}$$

where I = 4, …,10 are the internal indices of X^7. The above condition implies that the holonomy of X^7 has to be the maximal proper subgroup of $SO(7)$ which is G_2 (for a review, see for example[5]). Moreover, this G_2 manifold is a Ricci flat manifold obeying the equation of motion of the eleven-dimensional supergravity when the four-form field strength G and the gravitino is zero: these equations are simply the vacuum Einstein equations.

When the fluxes are present, *i.e.* $G\neq 0$ the seven dimensional X^7 is no longer a Ricci flat manifold and further implies that the vacua of M-theory in four dimensions are not supersymmetric. In other words the compactification in the presence of fluxes is a description of spontaneous supersymmetry breaking. Let us look closely this phenomenon. In compactification of eleven-dimensional supergravity, massless scalar in four dimensions can originate from either the metric or the C-field via[5])

$$C_{IJK} = \sum_{\mathrm{i}}\phi^{\,i}(x)\,\omega^i{}_{IJK}(y) + \ldots \tag{10}$$

where $\omega^i{}_{IJK}(y)$ form a basis of the harmonic three forms $\omega^i(y)$ on X^7, $i=1,\ldots,b_3(X^7)$ with $b_3(X^7)$ is the third Betti number of X^7, and the dots refer to further terms related to the gauge field $A_\mu(x)$ and a harmonic two forms. Each scalar $\phi^i(x)$ in (10) appears as the real component of the complex scalar z^i in a chiral multiplet. On the other hand, the corresponding imaginary components of the z^i describe massless fluctuations in the background metric on X^7 [8]). This can be seen as follows. For any metric of G_2 holonomy on X^7, there is a unique covariantly constant (hence closed and co-closed) three form Φ_{IJK}. This three form Φ_{IJK} belongs to the cohomology class [Φ] in $H^3\left(X^7;\mathrm{R}\right)$, and this assignment is invariant under diffeomorphisms of X^7. As it was shown in[9]), the moduli space of G_2 holonomy metrics on X^7, modulo diffeomorphisms isotopic to the identity, is a smooth manifold of dimension. Furthermore, near a point in the moduli space corresponding to the equivalence class of metrics associated to Φ_{IJK}, the moduli space is locally diffeomorphic to an open ball about [Φ] in $H^3\left(X^7;\mathrm{R}\right)$. These show that massless mode of the metric on X^7 can be parameterized as

$$\Phi_{IJK} = \sum_i s^i\omega^i_{IJK} \tag{11}$$

The s^i are presumed to fluctuate around some point away from the origin. Thus the s^i in (11) naturally combine with the c^i as $z^i = c^i + \mathrm{i}\,s^i$.

For compactification in the presence of fluxes, one can introduce the superpotential $W(z)$ which was proposed in[8,10]):

$$W(z) = \frac{1}{8\pi^2}\int\sqrt{g^{(7)}}\,\varepsilon^{IJKLI_1J_1K_1}\left(\frac{1}{2}C_{IJK} + \mathrm{i}\Phi_{IJK}\right)G^{\left(X^7\right)}_{LI_1J_1K_1}\,d^7x \tag{12}$$

where $g^{(7)}$ is determinant of the metric g_{IJ} on X^7 and $G^{\left(X^7\right)}_{LI_1J_1K_1}$ are internal fluxes on X^7. Under a field variation

$$C_{IJK} \to C_{IJK} + \delta C_{IJK} \quad \Phi_{IJK} \to \Phi_{IJK} + \delta\Phi_{IJK} \quad (13)$$

the superpotential varies as

$$W \to W + \frac{1}{8\pi^2} \int \sqrt{g^{(7)}} \varepsilon^{IJKLI_1J_1K_1} \left(\delta C_{IJK} + \mathrm{i}\delta\Phi_{IJK}\right) G^{\left(X^7\right)}_{LI_1J_1K_1}\, d^7x \quad (14)$$

whereas $d\Phi = 0$ and $dC = G$. We see that δW is linear in $\delta C + \mathrm{i}\delta\Phi$ and thus the superpotential W is holomorphic. Furthermore, the scalar potential V can be written in terms of the superpotential W as[11)]

$$V^{N=1} = e^{K/M_P^2}\left(g^{i\bar{j}} D_i W\, \overline{D}_{\bar{j}}\overline{W} - \frac{3}{M_P^2} W\overline{W}\right) \quad (15)$$

where $D_i W = \partial_i W + K_i W / M_P^2$, M_P is the Planck scale, and K is the Kaehler potential which is given by[8)]

$$K = -3\ln\left(\frac{1}{14\pi^2}\int\sqrt{-g^{(7)}}\,\Phi_{IJK}\Phi^{IJK} d^7x\right) \quad (16)$$

The necessary condition for broken supersymmetry in Minkowskian ground states is given by[11,12)]

$$\left\langle g^{i\bar{j}} D_i W\, \overline{D}_{\bar{j}}\overline{W}\right\rangle = 3\left\langle W\,\overline{W}\right\rangle \quad (17)$$

and further, a gravitino becomes massive after eating a massless spin-$\frac{1}{2}$ fermion. The mass of gravitino $m_\Psi = \left\langle e^{K/M_P^2}|W|\right\rangle$ determines the scale of supersymmetry breaking.

$$F_i\, F^i = g^{i\bar{j}} D_i W\, \overline{D}_{\bar{j}}\overline{W} \quad (18)$$

which induces the soft breaking terms parameterized by m and A. In the early universe it has been generally assumed that the soft parameters are of order m_Ψ (assuming hidden sector supersymmetry breaking). The terms in (18) is also assumed to be dominated in the inflationary epoch, *i.e.* as the source of the *inflaton* field[14)]. Furthermore, the presence of Yukawa couplings in the theory ensures the existence of four field interaction terms, and therefore gives an effective mass for z of

$$\delta L = \left(\rho / M_P^2\right) \phi^+\phi \quad (19)$$

where ϕ is a scalar field which spans the moduli space of our theory and corresponds to the flat direction of the scalar potential.

In order to get such a result, one has to set $D_i W \propto H\, M_P$ and $W \propto H M_P^2$. This implies

5. Summary and Outlook

In this paper we have studied the low energy limit of M-theory which can be viewed as an eleven dimensional supergravity. Two situations have been discussed to argue the existence of the M-theory by using the type IIA supergravity on S^1 and the heterotic $E_8 \times E_8$ supergravity on $S^1/\mathbf{Z}_2$. Furthermore, we have considered the four dimensional physics of eleven dimensional supergravity on G_2 manifold without and with fluxes. As it was shown, the latter is the realization of spontaneous $N = 1 \to N = 0$ supersymmetry breaking. We then applied this study to describe the inflationary epoch of our universe at a very early time.

Some problems remain. First, it is still unknown how M-theory can also be viewed as the strong coupling limit of type IIB, $SO(32)$ heterotic, and type I string theories. It could be that there is a general compactification to solve this problem. On the other hand, there might be a non-trivial relation between the scale of supersymmetry breaking and the cosmological constant. This relation should explain the smallness of the cosmological constant of our present universe.

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